Lesson 38: Combinations and Compositions of Functions

By the end of the lesson, we will be able to:

~ Understand combinations of functions.
~ Combine functions by addition, subtraction, and multiplication.
~ Evaluate functions for a given value.
~ Find $f(\underline{g(x)})$ - compositions of functions
~ Find $g(\underline{f(x)})$ - compositions of functions

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Operations can be done with functions.

When adding, subtracting, multiplying, or dividing functions, you must write the resulting expression in its simplest form. This is an easy process, but you have to understand the new notation and symbols.

$$(f+g)(x) \ \ means \ \ f(x)+g(x)$$

$$(f-g)(x) \ \ means \ \ f(x)-g(x)$$

$$(fg)(x) \ \ means \ \ f(x)\cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) \ \ means \ \ \frac{f(x)}{g(x)}$$

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Examples:

Find each combination for

$f(x) = x^2 - 2x + 4$ & $g(x) = 3x^2 - 1$

A.) $(f+g)(x) = f(x) + g(x)$

$= (x^2 - 2x + 4) + (3x^2 - 1)$

$= x^2 - 2x + 4 + 3x^2 - 1$

$\boxed{(f+g)(x) = 4x^2 - 2x + 3}$

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Examples:

Find each combination for

$$f(x) = x^2 - 2x + 4 \quad \& \quad g(x) = 3x^2 - 1$$

B.) $(f - g)(x) = f(x) - g(x)$

$= (x^2 - 2x + 4) - (3x^2 - 1)$

$= x^2 - 2x + 4 - 3x^2 + 1$

$$\boxed{(f-g)(x) = -2x^2 - 2x + 5}$$

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Examples:

Find each combination for

$$f(x) = x^2 - 2x + 4 \quad \& \quad g(x) = 3x^2 - 1$$

C.) $(fg)(x) = f(x) \cdot g(x)$

$= (x^2 - 2x + 4)(3x^2 - 1)$

$= 3x^4 - x^2 - 6x^3 + 2x + 12x^2 - 4$

$$\boxed{(fg)(x) = 3x^4 - 6x^3 + 11x^2 + 2x - 4}$$

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Examples:

Find each combination for

$f(x) = x^2 - 2x + 4$ & $g(x) = 3x^2 - 1$

D.) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-2x+4}{3x^2-1}$

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{x^2-2x+4}{3x^2-1}}$$

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Now that you can combine functions, lets learn to evaluate for a given value.

Evaluating combinations of functions can be done in 2 ways: First Way

1) Combine First

* Combine the functions → +, −, ×, ÷
* Simplify the new function
* Plug in the value

For example, to find $(f+g)(2)$, first find $(f+g)(x)$, then plug 2 into the new function.

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Evaluating combinations of functions can be done in 2 ways: **Second Way**

2) Evaluate First

$f(2)$, $g(2)$

* Plug the value into <u>each</u> function separately
* Combine the resulting values

For example, to find $(f+g)(2)$, first find $(f)(2)$ and $(g)(2)$ separately, then add the resulting values together.

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Examples:

Evaluate each combination for

$$f(x) = 4x - x^2 \quad \& \quad g(x) = x^2 + 1$$

1st method

A.) $(f + g)(2)$ (Solve by combining functions first.)

$(f+g)(x) = (4x-x^2) + (x^2+1)$

$= 4x - x^2 + x^2 + 1$

$(f+g)(x) = 4x+1$

$(f+g)(2) = 4(2)+1$

$= 8+1$

$\boxed{(f+g)(2) = 9}$

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Examples:

Evaluate each combination for

$$f(x) = 4x - x^2 \quad \& \quad g(x) = x^2 + 1$$

A.) $(f+g)(2)$

2nd way

(Solve by evaluating functions first.)

$f(2) = 4(2) - (2)^2$

$= 8 - 4$

$f(2) = 4$

$g(2) = (2)^2 + 1$

$= 4 + 1$

$g(2) = 5$

$(f+g)(2) = f(2) + g(2)$

$= 4 + 5$

$\boxed{(f+g)(2) = 9}$

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Examples:

Evaluate each combination for

$$f(x) = 4x - x^2 \quad \& \quad g(x) = x^2 + 1$$

2nd method

B.) $(f-g)(3)$

$f(3) = 4(3) - (3)^2$
$= 12 - 9$
$f(3) = 3$

$g(3) = (3)^2 + 1$
$= 9 + 1$
$g(3) = 10$

$(f-g)(3) = f(3) - g(3)$
$= 3 - 10$

$\boxed{(f-g)(3) = -7}$

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Examples:

Evaluate each combination for

$$f(x) = 4x - x^2 \quad \& \quad g(x) = x^2 + 1$$

C.) $(fg)(4)$

$f(4) = 4(4) - (4)^2$

$= 16 - 16$

$f(4) = 0$

$g(4) = (4)^2 + 1$

$= 16 + 1$

$g(4) = 17$

$(fg)(4) = (0)(17)$

$\boxed{(fg)(4) = 0}$

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Examples:

Evaluate each combination for

$$f(x) = 4x - x^2 \quad \& \quad g(x) = x^2 + 1$$

D.) $\left(\frac{f}{g}\right)(0)$

$f(0) = 4(0) - (0)^2$

$f(0) = 0$

$g(0) = 0^2 + 1$

$g(0) = 1$

$\left(\frac{f}{g}\right)(0) = \frac{0}{1} = 0$

$\boxed{\left(\frac{f}{g}\right)(0) = 0}$

Compositions of Functions

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We are now going to substitute functions into other functions.

For example $f(g(x))$, means that you substitute the function $g(x)$ in place of x in function f.

This is called a **composition** of functions.

Another way to write $f(g(x))$ is $(f \circ g)(x)$. Both of these are read "f of g of x".

* Note: $(f \circ g)(x) \neq (fg)(x)$.

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To do compositions of functions you follow these steps:

$f(g(x))$

1) Write down the outside function
2) Substitute the inside function for the variable
3) Simplify

Example: If $f(x) = 2x + 5$ and $g(x) = 4x + 1$, find $f(g(x))$.

1) $f(x) = 2x+5$

2) $f(g(x)) = 2(4x+1)+5$ (substituted $g(x)$ for x)

3) $f(g(x) = (8x+2)+5 = 8x+7$

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Examples: If $f(x) = 2x + 5$, $g(x) = 4x + 1$, and $h(x) = x^2$, find:

A. $g(f(x))$ $= 4(2x+5) + 1 = 8x + 20 + 1 = 8x + 21$

$$\boxed{g(f(x)) = 8x + 21}$$

B. $(f \circ h)(x)$ $= f(h(x)) = 2(x^2) + 5$

$$\boxed{(f \circ h)(x) = 2x^2 + 5}$$

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Examples: If $f(x) = 2x + 5$, $g(x) = 4x + 1$, and $h(x) = x^2$, find:

C. $(h \circ f)(x) = h(f(x)) = (2x+5)^2 = (2x+5)(2x+5)$

$= 4x^2 + 10x + 10x + 25 = 4x^2 + 20x + 25$

$\boxed{(h \circ f)(x) = 4x^2 + 20x + 25}$

D. $h(g(x))$

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Here is a real world example.

Let t(x) represent the tax added onto a purchase. Let $d(x)$ represent a discount taken from a purchase.
A store is offering a $10 discount on sale items.
Customers must pay 6.25% tax on purchases.

E. Write functions for t(x) and $d(x)$.

F. Explain the meaning of t($d(x)$).

G. Explain the meaning of $d(t(x))$.

Evaluating compositions of functions can be done in 2 ways:

1.) Find the compositions of the functions, then substitute the value.

or

2.) Substitute the value into the "inner" function and evaluate. Then take that answer and substitute into the "outer" function.

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Examples: For $f(x) = 2x + 5$, $g(x) = 4x + 1$, and $h(x) = x^2$, find:

1st

H. $f(g(3))$

$f(g(x)) = 2(4x+1) + 5 = 8x + 2 + 5 = 8x + 7$

$f(g(3)) = 8(3) + 7 = 24 + 7 = 31$

$\boxed{f(g(3)) = 31}$

2nd

I. $g(f(3))$

$f(3) = 2(3) + 5$
$= 6 + 5$
$f(3) = 11$

$g(f(3)) = g(11) = 4(11) + 1$
$= 44 + 1$
$g(11) = 45$

$\boxed{g(f(3)) = 45}$

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Examples: For $f(x) = 2x + 5$, $g(x) = 4x + 1$, and $h(x) = x^2$, find:

J. $(f \circ h)(5) = f(25) = 2(25) + 5$

$= 55$

$h(5) = (5)^2$

$= 25$

$\boxed{(f \circ h)(5) = 55}$

K. $(h \circ f)(5)$

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Examples: For $f(x) = 2x + 5$, $g(x) = 4x + 1$, and $h(x) = x^2$, find:

L. $h(g(2))$

M. $f(f(4))$

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Let t(x) represent the tax added onto a purchase. Let $d(x)$ represent a discount taken from a purchase. A store is offering a $10 discount on sale items. Customers must pay 6.25% tax on purchases. This means $t(x) = 1.0625x$ and $d(x) = x - 10$

R. Find $t(50)$ and explain what it means.

S. Find $d(50)$ and explain what it means.

T. Find t$(d(50))$ and explain what it means.

U. Find $d(t(50))$ and explain what it means.

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~ Understand combinations of functions.

~ Combine functions by addition, subtraction, and multiplication.

~ Evaluate functions for a given value.

~ Find $f(g(x))$ - compositions of functions

~ Find $g(f(x))$ - compositions of functions

Can you?

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Homework:

Assignment 38

& Review for test

A-day - Test on Wed, Feb 15

B-day - Test on Tues, Feb 14th

(next time!)