

Lesson # 40: Vertex form of a Parabola

$$\underline{x^2}$$

By the end of this lesson you will be able to:

- ~ Write an equation for a graphed function
- ~ Convert equations to it's Vertex Form
- ~ Graph equations from Vertex Form

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Writing Equations From Graphs

We can determine a function's equation by using the vertex and stretch. Steps for writing an equation from a given graph:

1. Find the vertex, (h,k) .
2. Plug in the h and k values into the vertex formula:
$$y = a(x - h)^2 + k.$$
3. Now pick another point, (x,y) , from the graph.
4. Plug in the values for x and y into your new equation and then solve for a .
5. Plug in the value for a into your equation from step 2.

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$$y = a(x-h)^2 + k$$

Writing Equations From Graphs

Find the equation for the function.

vertex: $(-2, -1)$

$$h = -2, k = -1$$

$$y = a(x+2)^2 - 1$$

pick: $(-3, 0)$

$$0 = a(-3+2)^2 - 1$$

$$0 = a(-1)^2 - 1$$

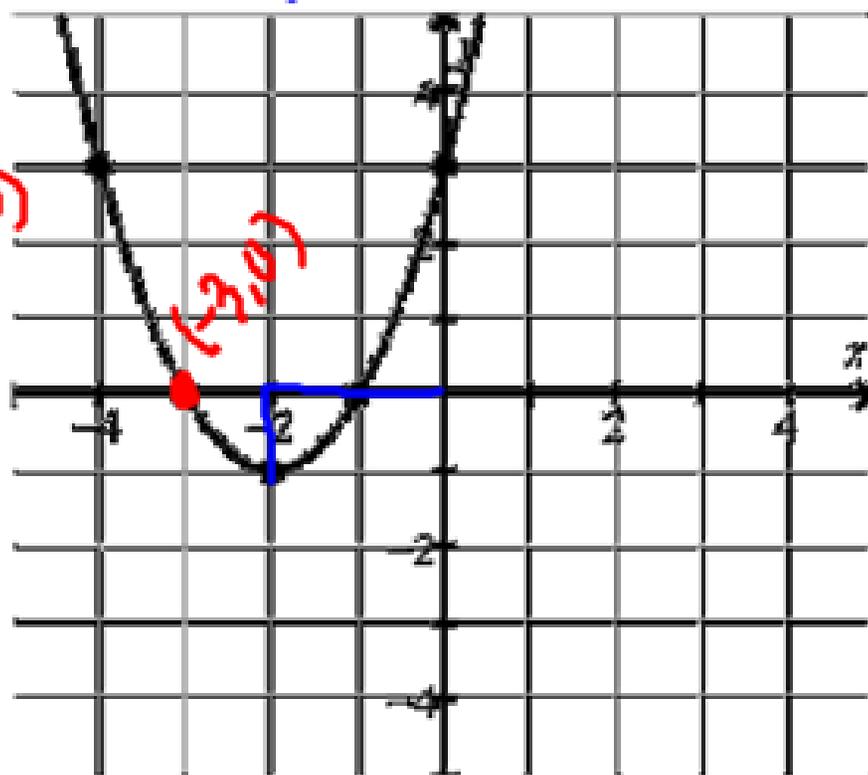
$$0 = 1a - 1$$

$$\begin{array}{r} +1 \\ \hline 1 = 1a \end{array}$$

$$1 = 1a$$

$$a = 1$$

$$y = (x+2)^2 - 1$$



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Writing Equations From Graphs

Find the equation for the function.

vertex: $(2, 4)$

$$y = a(x - 2)^2 + 4$$

$$0 = a(0 - 2)^2 + 4$$

$$0 = a(-2)^2 + 4$$

$$0 = 4a + 4$$

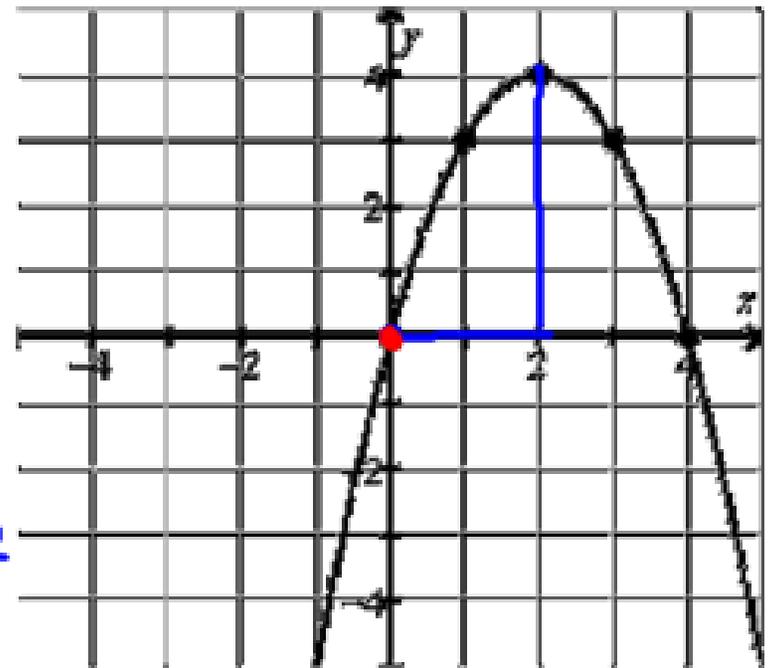
$$\begin{array}{r} -4 \qquad \qquad -4 \\ \hline \end{array}$$

$$\frac{-4}{4} = \frac{4a}{4}$$

$$a = -1$$

$$y = -(x - 2)^2 + 4$$

Pick: $(0, 0)$



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Writing Equations From Graphs

Find the equation for the function.

Vertex is $(2,0)$ and goes through the point $(3,4)$

$$y = a(x-2)^2 + 0$$

$$4 = a(3-2)^2$$

$$4 = a(1)^2$$

$$4 = 1a$$

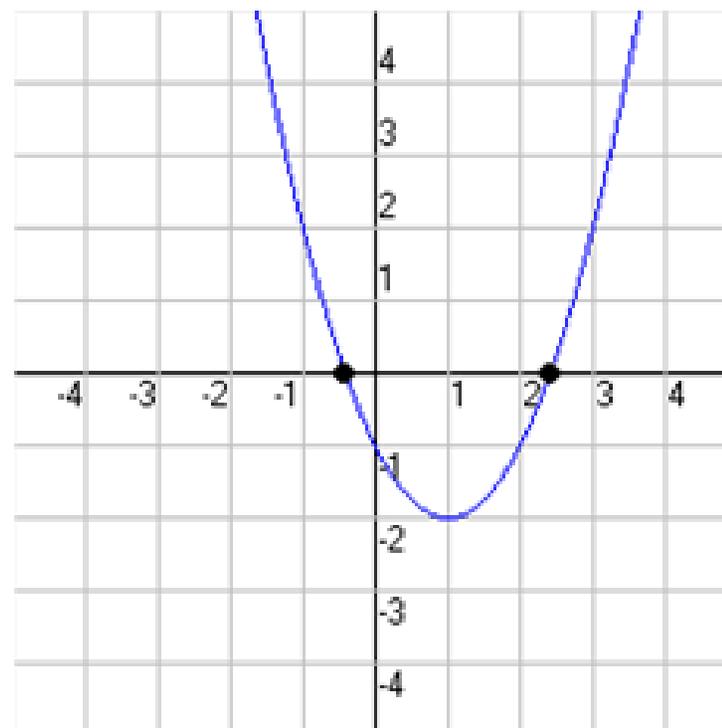
$$4 = a$$

$$y = 4(x-2)^2$$

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Writing Equations From Graphs

Find the equation for the function.



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Converting to Vertex Form

How do we graph a quadratic equation by hand that is not in vertex form?

This is very difficult to do, so instead we will convert the original equation into vertex form.

$$y = a(x-h)^2 + k$$

Remember: The quadratic function form $f(x) = ax^2 + bx + c$

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Finding the vertex:

$$h = \frac{-b}{2a}$$

We will find the x-coordinate of the vertex by using $\frac{-b}{2a}$.

We will find the corresponding y-value of the vertex by plugging $\frac{-b}{2a}$, into our equation and solving for y. $k = f\left(\frac{-b}{2a}\right)$

Now we have our VERTEX as $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

After we've found our vertex we will use it in our vertex form and we will use the original a from $f(x) = \underline{a}x^2 + bx + c$.

So we have $y = \underline{a}\left(x - \frac{-b}{2a}\right)^2 + f\left(\frac{-b}{2a}\right)$.

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Converting to Vertex Form

Example: Write each equation in vertex form.

$$f(x) = 2x^2 + 12x + 13 \quad \underline{a=2} \quad b=12 \quad c=13 \quad a=2$$

$$h = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3$$

$$h = -3$$

$$k = -5$$

$$\begin{aligned} k = f(-3) &= 2(-3)^2 + 12(-3) + 13 \\ &= 2(9) - 36 + 13 \\ &= 18 - 36 + 13 \\ &= -18 + 13 \\ k &= -5 \end{aligned}$$

vertex: $(-3, -5)$

$$f(x) = 2(x+3)^2 - 5$$

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Converting to Vertex Form

Example: Write each equation in vertex form.

$$f(x) = x^2 + 6x + 2 \quad a=1 \quad b=6 \quad c=2$$

$$h = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$$

$$k = f(-3) = (-3)^2 + 6(-3) + 2$$

$$= 9 - 18 + 2$$

$$= -9 + 2$$

$$k = -7$$

$$a = 1$$

$$h = -3$$

$$k = -7$$

Vertex: $(-3, -7)$

$$f(x) = (x+3)^2 - 7$$

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Converting to Vertex Form

Example: Write each equation in vertex form.

$$f(x) = -x^2 + 4x + 2 \quad a = -1 \quad b = 4 \quad c = 2$$

$$h = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$$

$$k = f(2) = -(2)^2 + 4(2) + 2$$

$$= -4 + 8 + 2$$

$$= 4 + 2$$

$$k = 6$$

$$a = -1$$

$$h = 2$$

$$k = 6$$

vertex: (2, 6)

$$f(x) = -(x-2)^2 + 6$$

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Graphing Quadratics

To graph:

1. Convert to Vertex Form.
2. Plot the Vertex.
3. Use a "t" chart to find two points to the left and two points to the right of the vertex.
4. Plot the points and connect the dots. (Remember arrows!)

vertex.

x	y
-2	
-1	
0	0
1	
2	

{

x	y
-6	
-5	
-4	2
-3	
-2	

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Graphing Quadratics

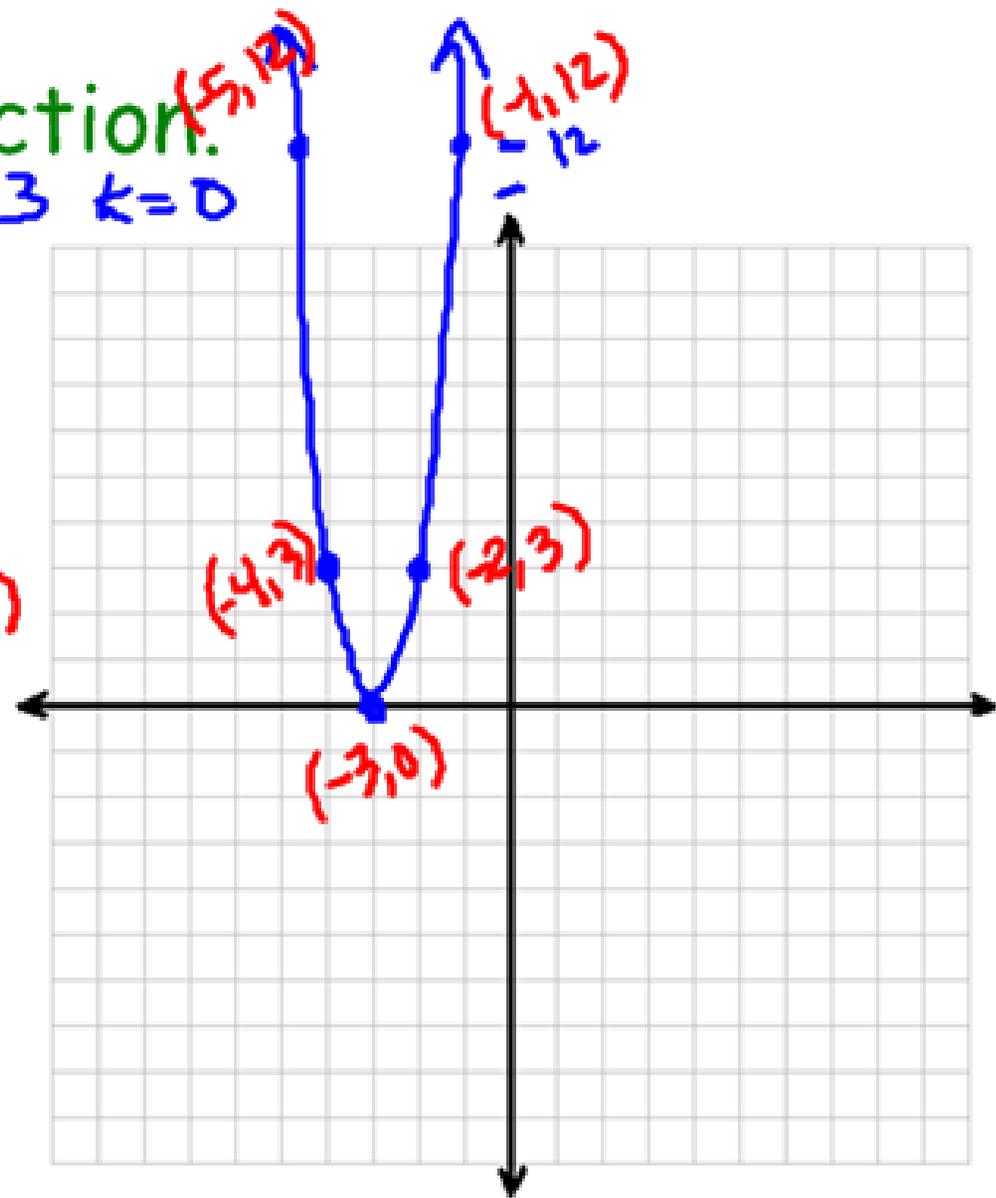
Ex 1: Graph the function.

$$h = -3 \quad k = 0$$

$$f(x) = 3(x + 3)^2$$

vertex: $(-3, 0)$

x	$y = 3(x+3)^2$
(-5)	$12 = 3(-5+3)^2 = 3(-2)^2 = 3(4)$
(-4)	$3 = 3(-4+3)^2 = 3(-1)^2 = 3$
(-3)	0
(-2)	$3 = 3(-2+3)^2 = 3(1)^2 = 3$
(-1)	12



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Ex 1: Graph the function. (WORK)

$$f(x) = 3(x + 3)^2$$

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Graphing Quadratics

Ex 2: Graph the function.

$$f(x) = x^2 + 6x + 2$$

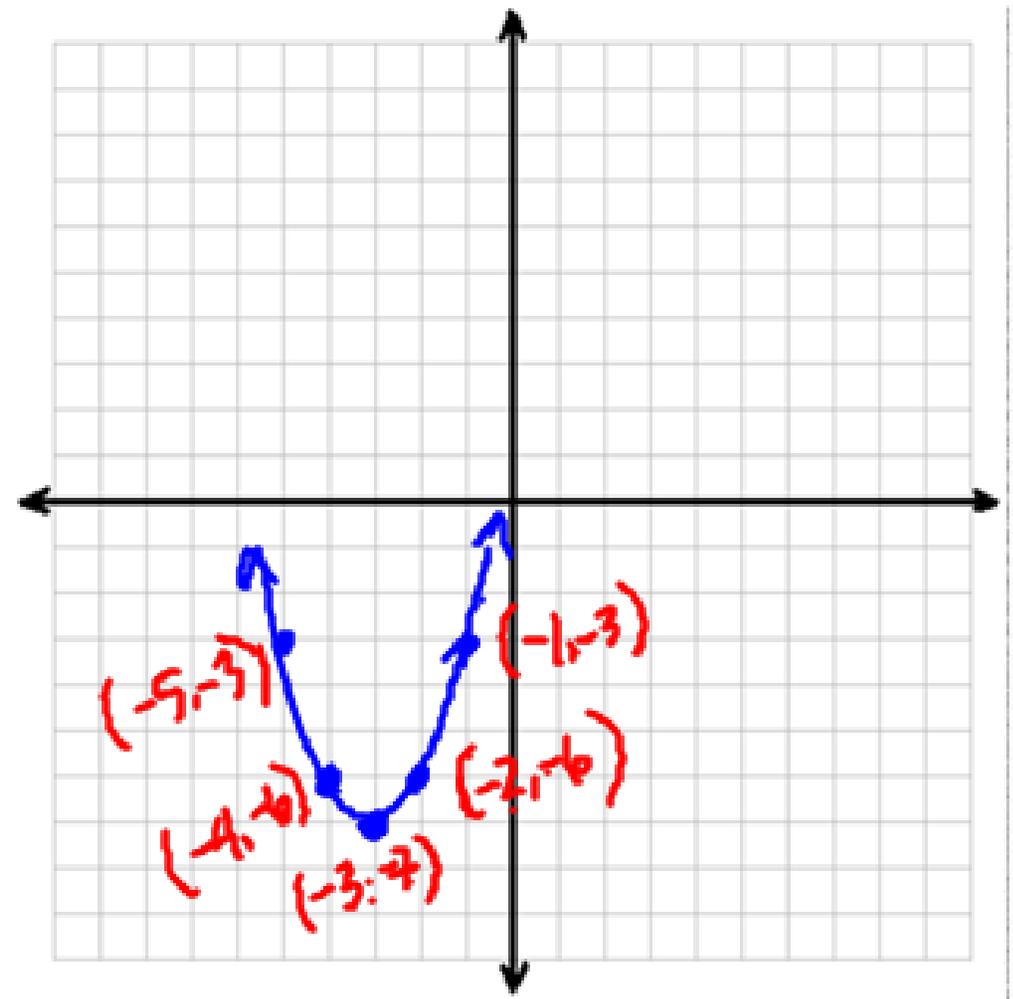
$$f(x) = (x+3)^2 - 7$$

Vertex: $(-3, -7)$

x	$y = (x+3)^2 - 7$
-5	-3
-4	-6
-3	-7
-2	-6
-1	-3

$v: \begin{cases} (-5, -3) \\ (-4, -6) \\ (-3, -7) \\ (-2, -6) \\ (-1, -3) \end{cases}$

$\begin{cases} (-5, -3) = (-5+3)^2 - 7 = (-2)^2 - 7 = \\ (-4, -6) = (-4+3)^2 - 7 = (-1)^2 - 7 \end{cases}$



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Ex 2: Graph the function. (WORK)

$$f(x) = x^2 + 6x + 2$$

a=1, b=6, c=2

$$h = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2} = -3$$

$$\begin{cases} a = 1 \\ h = -3 \\ k = -7 \end{cases}$$

$$k = f(-3) = (-3)^2 + 6(-3) + 2$$

vertex: (-3, -7)

$$= 9 - 18 + 2$$

$$= -9 + 2$$

$$k = -7$$

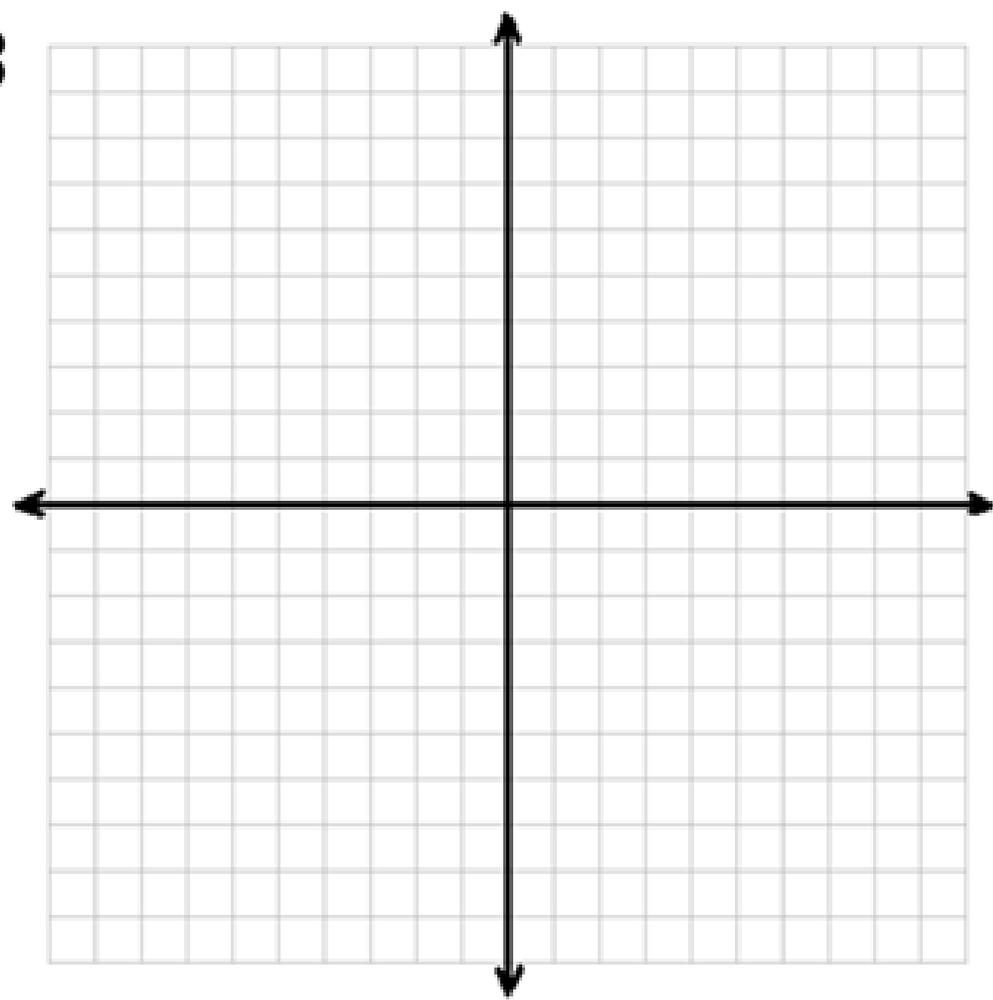
$$f(x) = (x+3)^2 - 7$$

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Graphing Quadratics

Ex 3: Graph the function.

$$f(x) = -5x^2 - 40x - 8$$



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Ex 3: Graph the function. (WORK)

$$f(x) = -5x^2 - 40x - 80$$

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Can you?

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Homework:

Instructions: Ignore the "axis of symmetry".

Assignment 40