Lesson #41: Inverse of Functions

By the end of this lesson, you will be able to:

- ~ Find the inverse of an equation or graph
- ~ Determine whether a function is one-to-one
- ~ Determine whether functions are inverses of each other

Lesson #41: Inverse of Functions

Inverses of Relations:

The INVERSE of a relation can be found by interchanging the x and y values.

Remember: For a relation to be a function, it must have only one y value to every x value.

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Inverses of Relations:

Example: Find the inverse of each & determine whether the inverse is a function.

A. $\{(1, 3), (1, -1), (1, -3), (1, 1)\}$

Inverse:

Inverse is a function? Yes No

B. $\{(6, 11), (-2, 7), (0, 3), (-5, 3)\}$

Inverse:

Inverse is a function? Yes No

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Inverses of Graphs:

Because we can interchange the x and y values of each point, and inverse graph can be drawn.

Two inverse graphs will be a reflection of each other across the diagonal line $y = x$.

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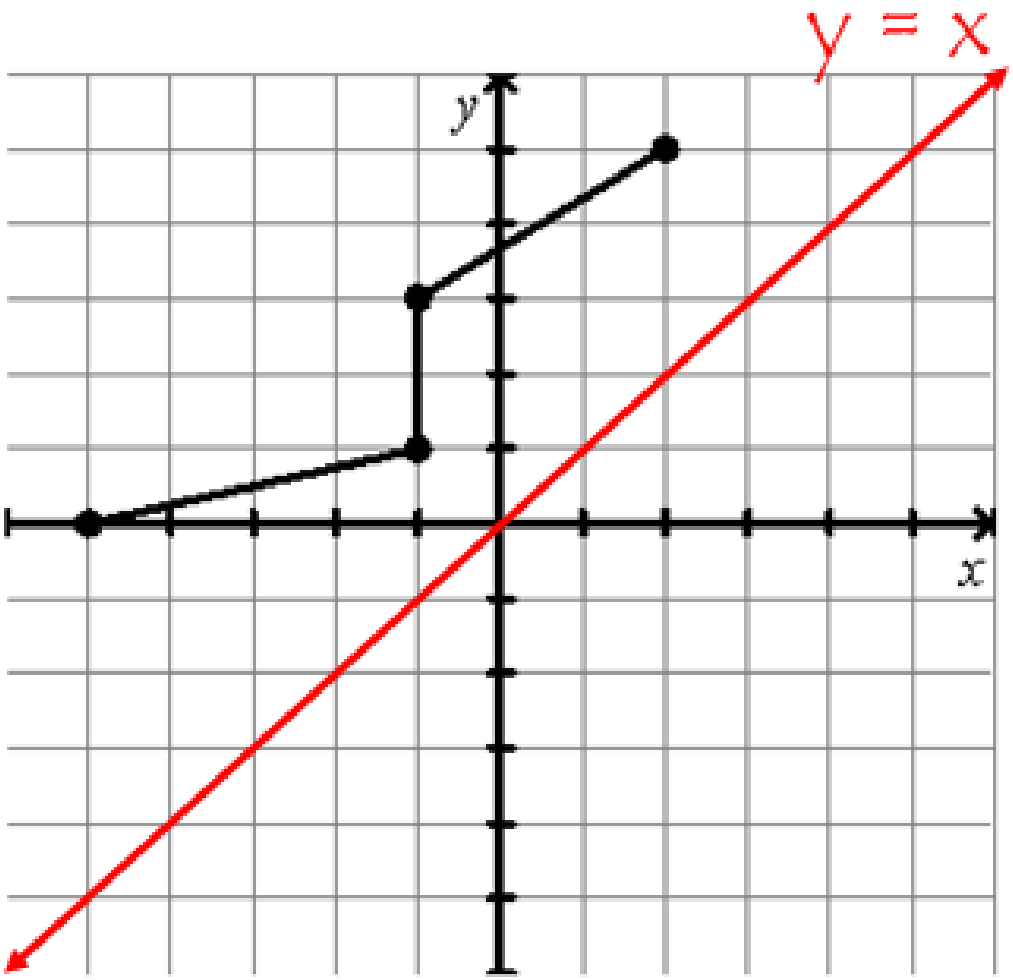
Inverses of Graphs:

Example: Draw each inverse and decide whether the inverse is a function.



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Inverses of Graphs:

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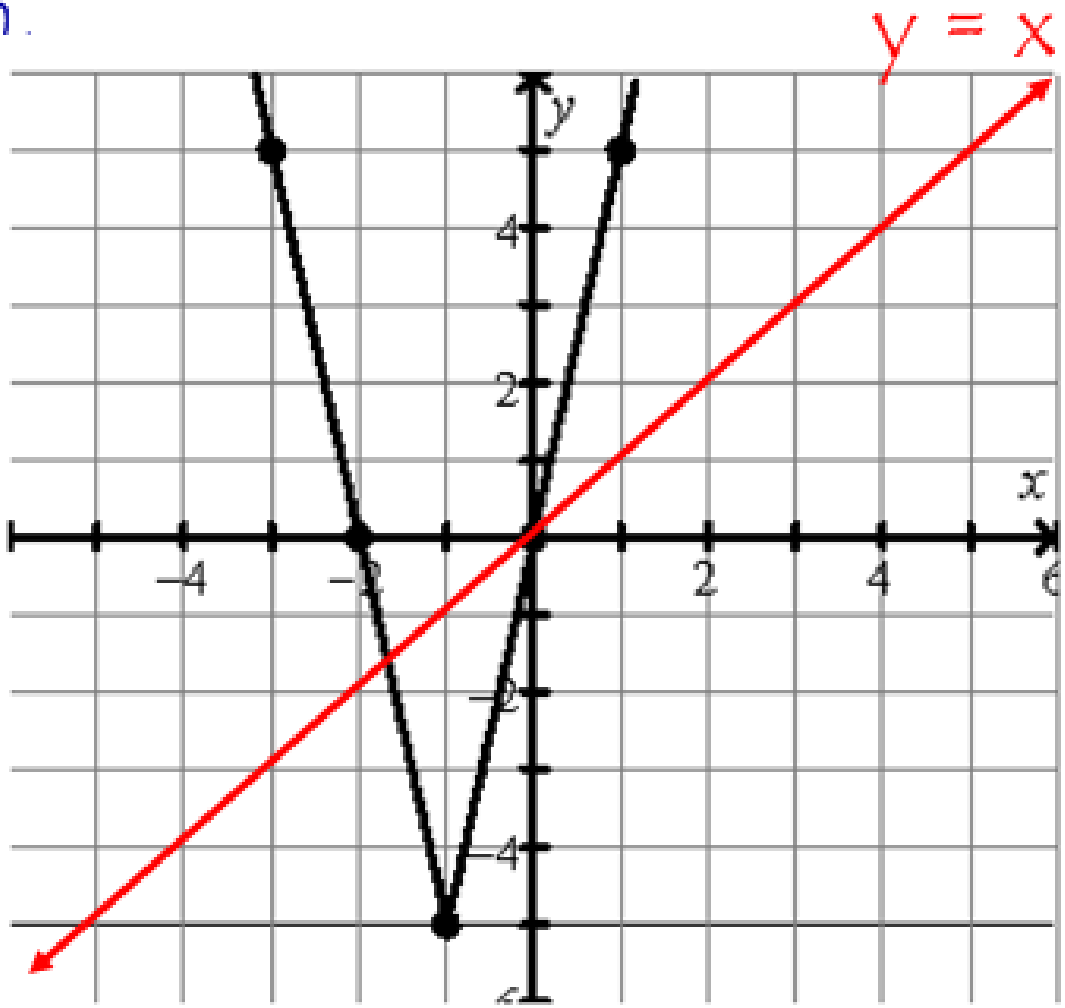

Lesson #41: Inverse of Functions

One-to-One Functions:

Functions are called one-to-one if every x-value is paired with exactly one y-value, and vice-versa.

On a graph, this means that the function would pass the VERTICAL LINE test and the HORIZONTAL LINE Test.

Note: A function that is NOT one-to-one, does NOT have an INVERSE.

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One-to-One Functions:

Examples: Graph the following functions on the calculator to determine whether each is a one-to-one function.

$$f(x) = \frac{5x+3}{2}$$

One-to-one? Yes No

$$h(x) = 5^x - 3$$

One-to-one? Yes No

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Inverses of functions:

For a pair of functions to be inverse functions, the compositions of both functions must result in the identity function y = x.

$f(x)$ and $g(x)$ are **inverse functions** if and only if $f(g(x)) = x$ and $g(f(x)) = x$

Note: The inverse of f(x) can be written using the notation $f^{-1}(x)$.

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Inverses of functions:

Example: Determine whether each pair of functions are inverse functions.

$$f(x) = \frac{2}{3}x - 4$$

$$g(x) = \frac{3}{2}x + 6$$

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Inverses of functions:

Example: Determine whether each pair of functions are inverse functions.

$$f(x) = 4x - 7$$
$$g(x) = -4x + 7$$

Finding Inverses Algebraically

To find the inverse of a function algebraically:

1. Interchange the x and y variables in the equation. (Replace f(x) with y.)

2. Solve for the new y.

3. If the inverse is a function, change the y to $f^{-1}(x)$. If it is not a function leave it as y.

Finding Inverses Algebraically

Example: Determine whether the function is one-to-one. If it is, find the inverse function.

$$f(x) = 4x - 2$$

Finding Inverses Algebraically

Example: Determine whether the function is one-to-one. If it is, find the inverse function.

$$h(x) = x^4 - 3x^3$$

Finding Inverses Algebraically

Example: Determine whether the function is one-to-one. If it is, find the inverse function.

$$h(x) = \sqrt{3x - 5}$$

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Can you:

~ Find the inverse of an equation or graph
~ Determine whether a function is one-to-one
~ Determine whether functions are inverses of each other

???

~Homework~

Assignment 41