Lesson #41: Inverse of Functions

By the end of this lesson, you will be able to:

- ~ Find the inverse of an equation or graph
- ~ Determine whether a function is one-to-one
- ~ Determine whether functions are inverses of each other

Lesson #41: Inverse of Functions

Inverses of Relations:

The INVERSE of a relation can be found by interchanging the x and y values.

Remember: For a relation to be a function, it must have only one y value to every x value.

Lesson #41: Inverse of Functions

Inverses of Relations:

Example: Find the inverse of each & determine whether the inverse is a function.

A. {(1, 3), (1, −1), (1, −3), (1, 1)}

Inverse: $\{(3,1),(-1,1)(-3,1)(1,1)\}$

Inverse is a function? (Yes) No

B. {(6, 11), (−2, 7), (0, 3), (−5, 3)}

Inverse: $\{(11,6),(7,-2),(3,0),(3,-5)\}$

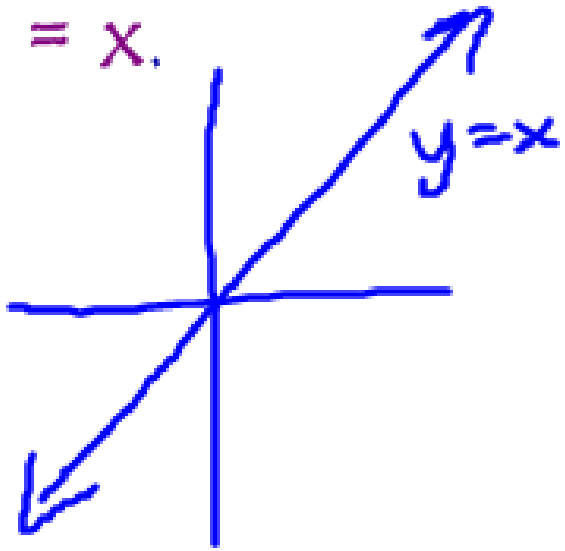
Inverse is a function? Yes (No)

Lesson #41: Inverse of Functions

Inverses of Graphs:

Because we can interchange the x and y values of each point, and inverse graph can be drawn.

Two inverse graphs will be a reflection of each other across the diagonal line y = x.



Lesson #41: Inverse of Functions

Inverses of Graphs:

Example: Draw each inverse and decide whether the inverse is a function.

flip x and y

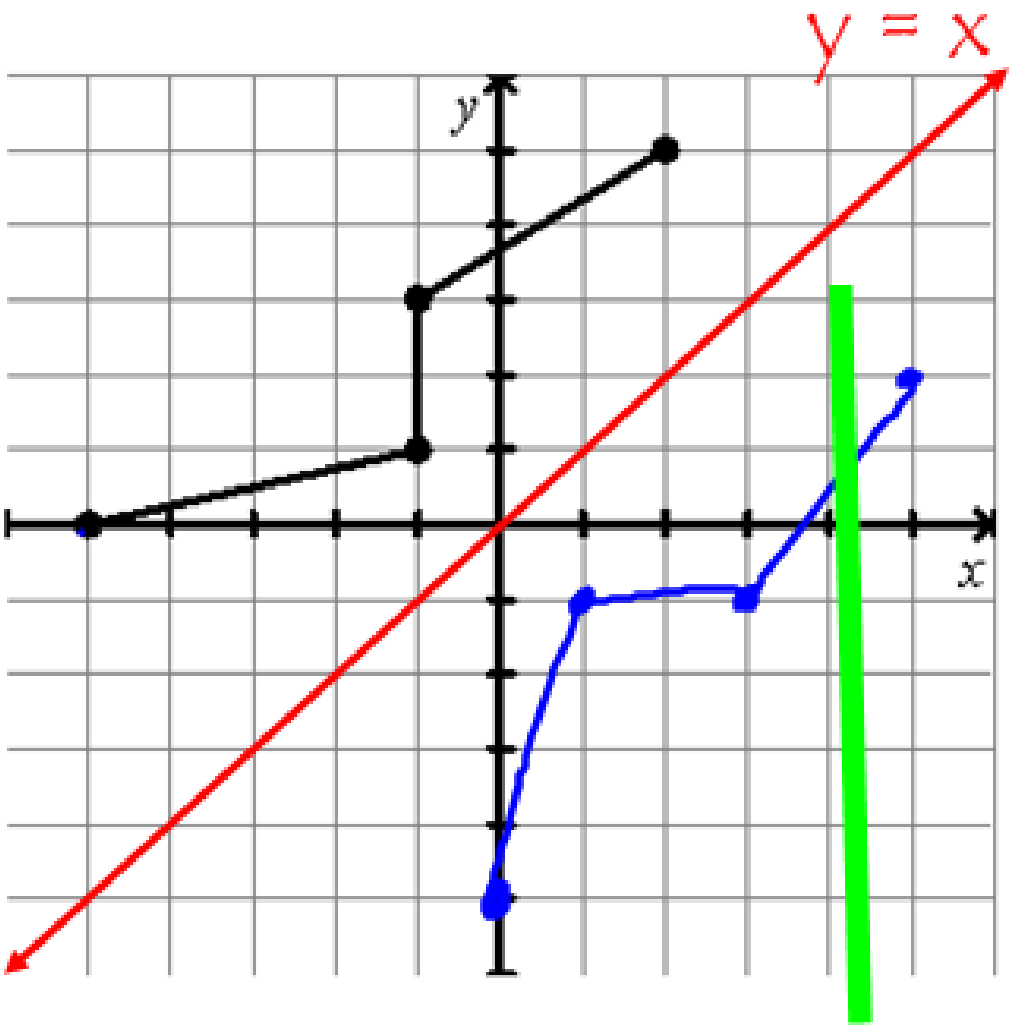
$(-5, 0) \rightarrow (0, -5)$

$(-1, 1) \rightarrow (1, -1)$

$(-1, 3) \rightarrow (3, -1)$

$(2, 5) \rightarrow (5, 2)$

Inverse is a function



Lesson #41: Inverse of Functions

Inverses of Graphs:

Example: Draw each inverse and decide whether the inverse is a function.

$(-3,5) \rightarrow (5,-3)$

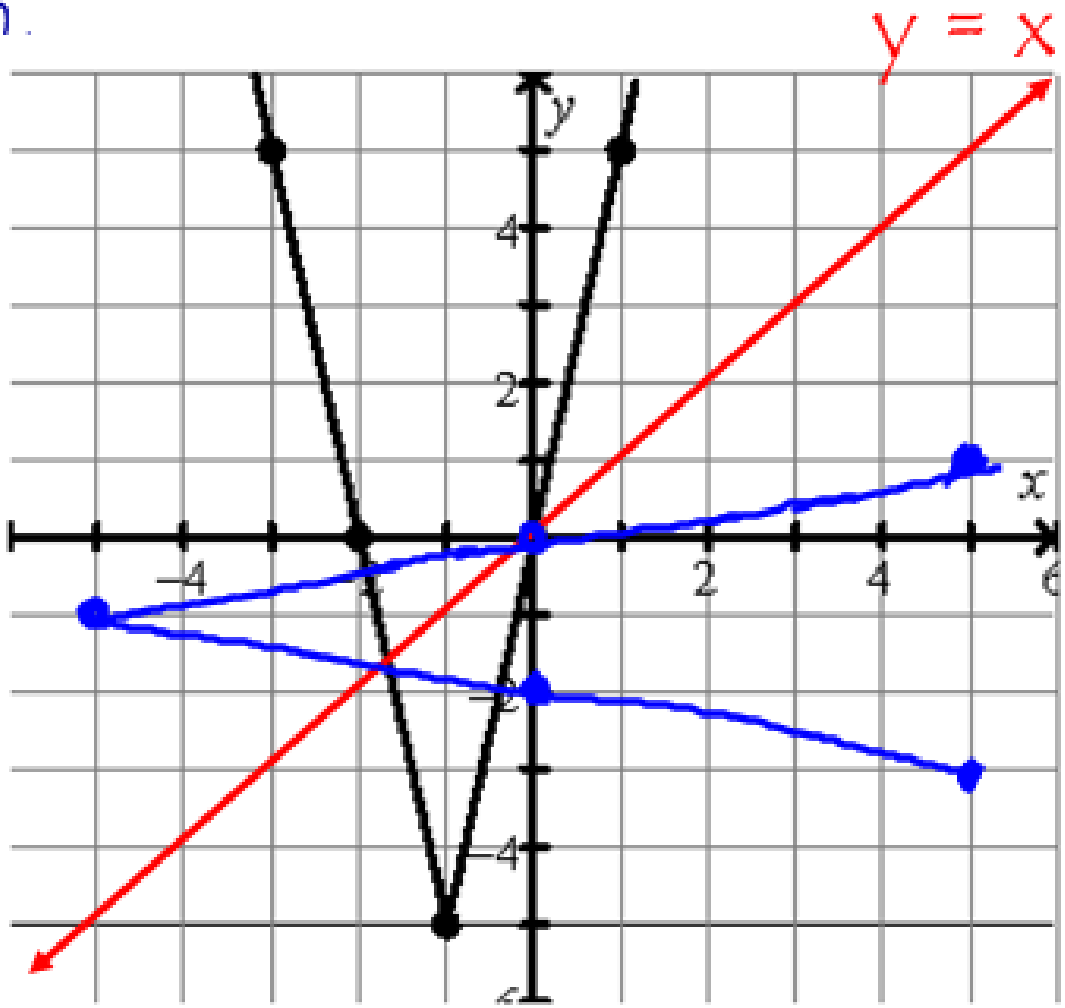
$(-2,0) \rightarrow (0,-2)$

$(-1,-5) \rightarrow (-5,-1)$

$(0,0) \rightarrow (0,0)$

$(1,5) \rightarrow (5,1)$

not a function



Lesson #41: Inverse of Functions

One-to-One Functions:

Functions are called one-to-one if every x-value is paired with exactly one y-value, and vice-versa.

On a graph, this means that the function would pass the VERTICAL LINE test and the HORIZONTAL LINE Test.

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Note: A function that is NOT one-to-one, does NOT have an INVERSE.

One-to-One Functions:

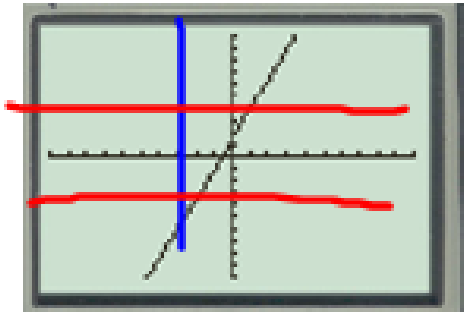
Examples: Graph the following functions on the calculator to determine whether each is a one-to-one function.

$$f(x) = \frac{5x+3}{2}$$

One-to-one? (Yes) No

VLT✓

HLT✓

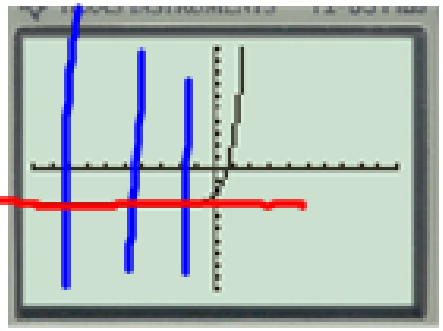


$$h(x) = 5^x - 3$$

One-to-one? Yes (No)

VLT✓

HLT



Lesson #41: Inverse of Functions

Inverses of functions:

For a pair of functions to be inverse functions, the compositions of both functions must result in the identity function y = x.

$f(x)$ and $g(x)$ are **inverse functions** if and only if $f(g(x)) = x$ and $g(f(x)) = x$

Note: The inverse of f(x) can be written using the notation $f^{-1}(x)$.

Inverses of functions:

Example: Determine whether each pair of functions are inverse functions

$$f(x) = \frac{2}{3}x - 4$$

$$g(x) = \frac{3}{2}x + 6$$

$f(g(x)) = \frac{2}{3}\left(\frac{3}{2}x + \frac{6}{1}\right) - 4$

$= \frac{2}{3} \cdot \frac{3}{2}x + \frac{2}{3} \cdot \frac{6}{1} - 4$

$= x + 4 - 4$

$f(g(x)) = x$

$g(f(x)) = \frac{3}{2}\left(\frac{2}{3}x - 4\right) + 6$

$= \frac{3}{2} \cdot \frac{2}{3}x - \frac{3}{2} \cdot \frac{4}{1} + 6$

$= x - 6 + 6$

$g(f(x)) = x$

yes, inverses

Inverses of functions:

Example: Determine whether each pair of functions are inverse functions.

$$f(x) = 4x - 7$$
$$g(x) = -4x + 7$$

$f(g(x)) = 4(-4x+7) - 7$

$= -16x + 28 - 7$

$f(g(x)) = -16x + 21 \neq x$ so

not inverses

Lesson #41: Inverse of Functions

Finding Inverses Algebraically

To find the inverse of a function algebraically:

one-to-one?

1. Interchange the x and y variables in the equation. (Replace f(x) with y.)

2. Solve for the new y.

3. If the inverse is a function, change the y to $f^{-1}(x)$. If it is not a function leave it as y.

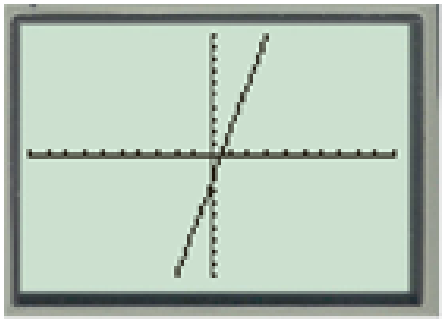
Lesson #41: Inverse of Functions

Finding Inverses Algebraically

Example: Determine whether the function is one-to-one. If it is, find the inverse function.

$f(x) = 4x - 2$ one-to-one? yes ✓

$y = 4x - 2$



Inverse: $x = 4y - 2$

$+2 \quad +2$

$\frac{x+2}{4} = \frac{4y}{4}$

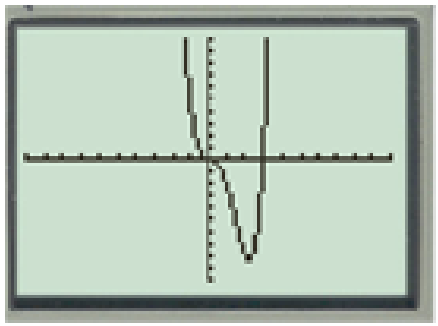
$y = \frac{x+2}{4}$

$f^{-1}(x) = \frac{x+2}{4}$

Finding Inverses Algebraically

Example: Determine whether the function is one-to-one. If it is, find the inverse function.

$$h(x) = x^4 - 3x^3$$



one-to-one? no

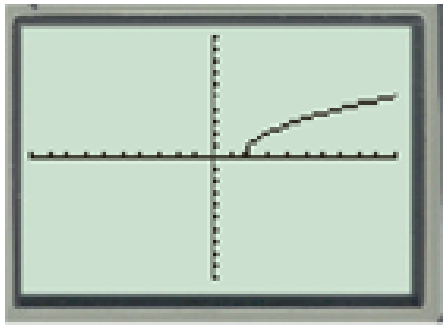
not one-to-one
so no inverse

Finding Inverses Algebraically

Example: Determine whether the function is one-to-one. If it is, find the inverse function.

$h(x) = \sqrt{3x - 5}$ one-to-one? yes ✓

$y = \sqrt{3x-5}$



Inverse: $(x = \sqrt{3y-5})^2$

$x^2 = 3y - 5$

$+5 \quad +5$

$x^2 + 5 = 3y$

$\frac{x^2+5}{3} = \frac{3y}{3}$

$y = \frac{x^2+5}{3}$

$h^{-1}(x) = \frac{x^2+5}{3}$

Lesson #41: Inverse of Functions

Can you:

~ Find the inverse of an equation or graph
~ Determine whether a function is one-to-one
~ Determine whether functions are inverses of each other

???

~Homework~

Assignment 41