Lesson 43: Exponential Functions

Objectives:

- ~ Graph Exponential equations and functions
  - * Horizontal Shift
  - * Vertical Shift
  - * Reflections
- ~ Evaluate Exponential expressions
- ~ Solve story problems

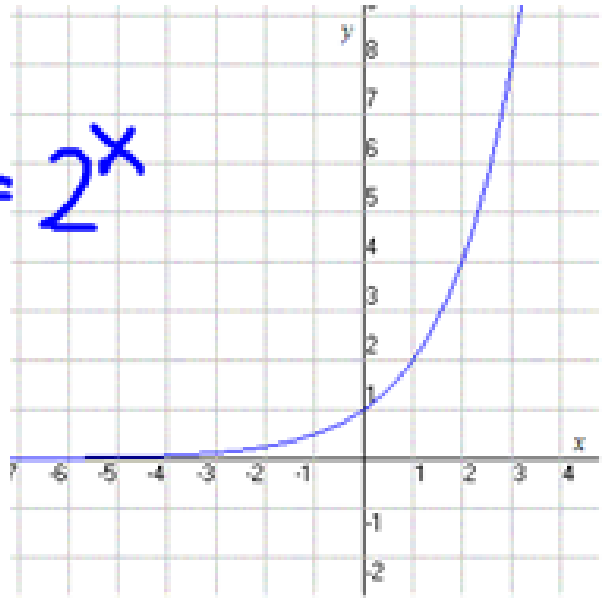
Lesson 43: Exponential Functions

In quadratic functions, x^2, the base x is variable, and the exponent 2 is constant.

However, In **exponential functions**, the base is constant and the exponent is variable. The exponential parent function is $y = b^x$ where b is a positive number other than 1.

Example of an exponential graph...

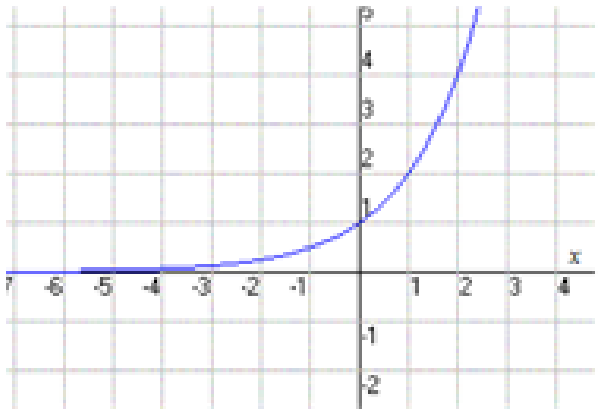
$y = 2^x$



Lesson 43: Exponential Functions

Exponential graphs level off and approach a line called an **asymptote**.

- For $y = b^x$, the asymptote is the x-axis, which is the horizontal line $y = 0$
- Since the graph never quite levels off completely, the **range** for $y = b^x$ is $y > 0$
- Since the graph goes outward forever in both directions, the **domain** is *always* $\mathbb{R}$



Special Points for $y = b^x$:

The first special point is $(0,1)$

The next special point is $(1,b)$

Other points:

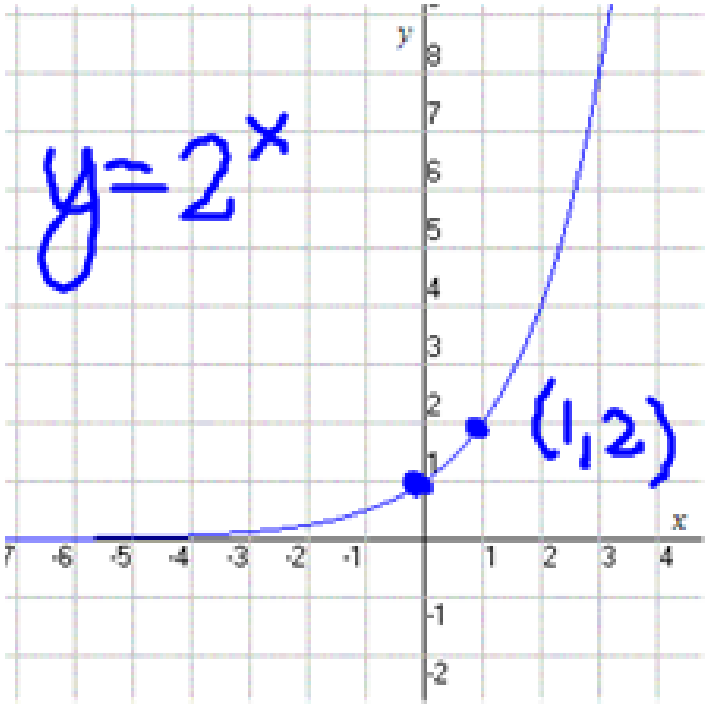
$(2, b^2)$

$(3, b^3)$

$(4, b^4)$

$\vdots$

$y = b^x$



Lesson 43: Exponential Functions

Translations:

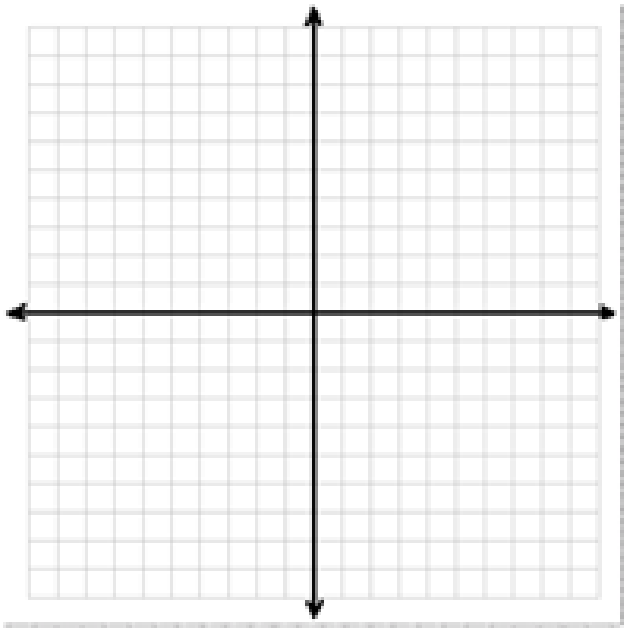
$y = b^{x-h} + k$

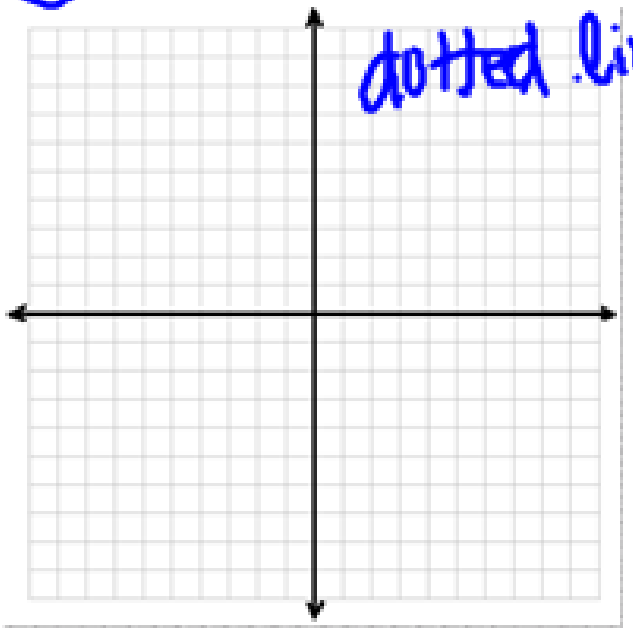
$y = b^{(x-h)} + k$

h affects horizontal (left/right) shift

k affects vertical (up/down) shift

Horizontal asymptote: $y = k$ graph w/ dotted line



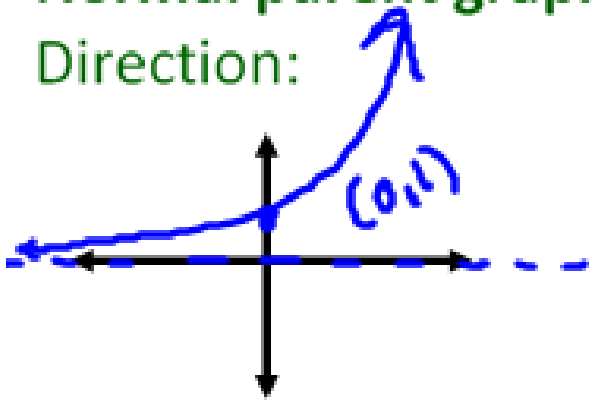


Reflections

$y = b^x$

Normal parent graph

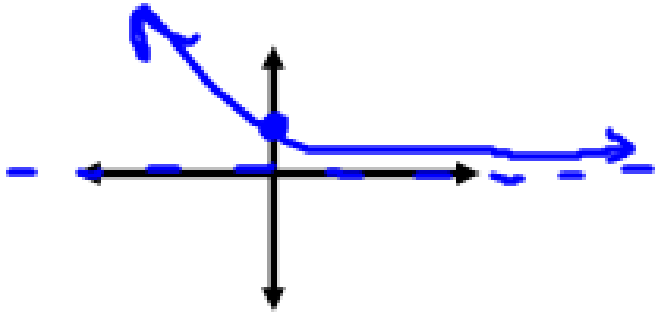
Direction:



$y = b^{-x}$

Flips the graph over y-axis

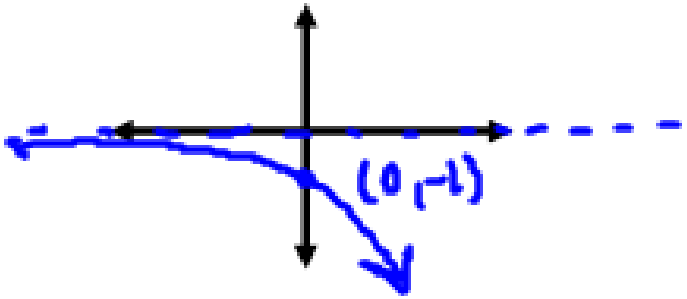
Direction: mult x by -1



$y = -b^x$

Flips the graph over x-axis

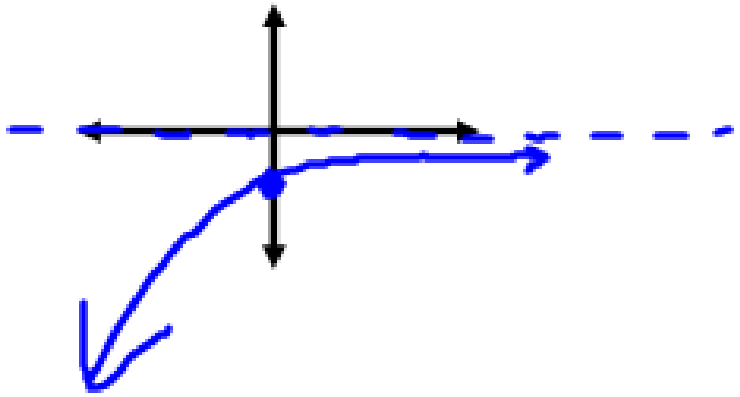
Direction: mult y by -1



$y = -b^{-x}$

Flips the graph over x & y axis

Direction: mult y & x by -1



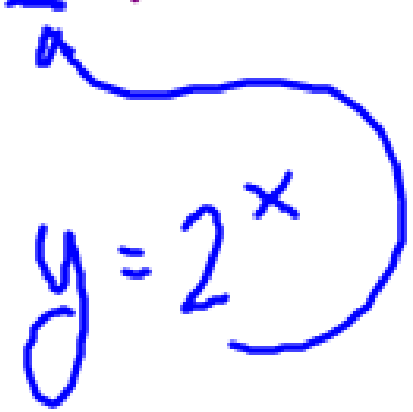
Steps to Graph an Exponential Function:

1. Identify and graph the **horizontal asymptote (HA)**. $y = k$
2. Write down special points (0, 1) (1, b). dotted line
3. Add the "h" value to the X's in your special points.
4. If "a" is negative (outside parentheses), make the "y" value in the special points negative.
   If "x" is negative (inside parentheses), make the "x" value in the special points negative.
   ~~ **This is our reflection step.** ~~
5. Add the "k" value to the Y's in your special points.
6. Plot the points and connect the dots.
   (Remember arrows!)
   The graph will always go horizontally along the asymptote.

Remember:

- Graph the first special point **1 space** from the asymptote.

- Graph the second special point **b spaces** from the asymptote.



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$y = b^{(x-h)} + k$ HA: $y = k$

Example 1:

$f(x) = 3^x$ h = 0 K = 0

HA: y = 0

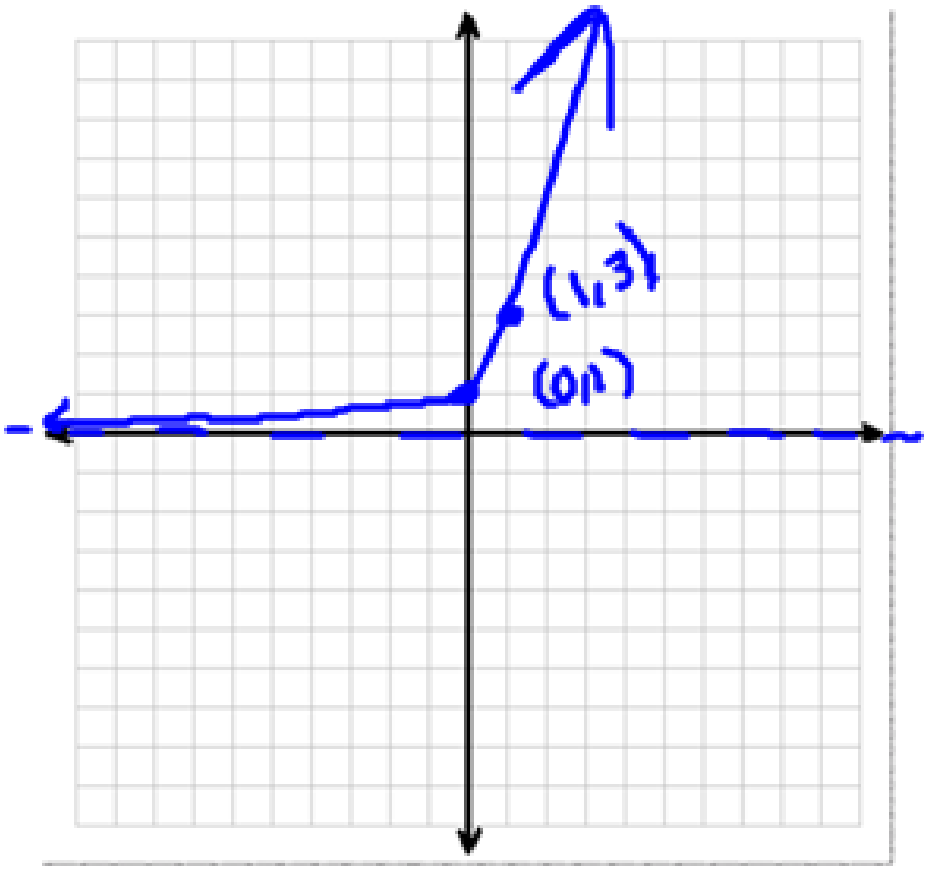
Domain: $\mathbb{R}$

Range: $y > 0$

SP

(0,1)

(1,3)



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Example 2:

$y = b^{(x-h)} + k$

$g(x) = -5^{x-2}$

$h=2$ $k=0$ $a=-1$

HA: $y=0$

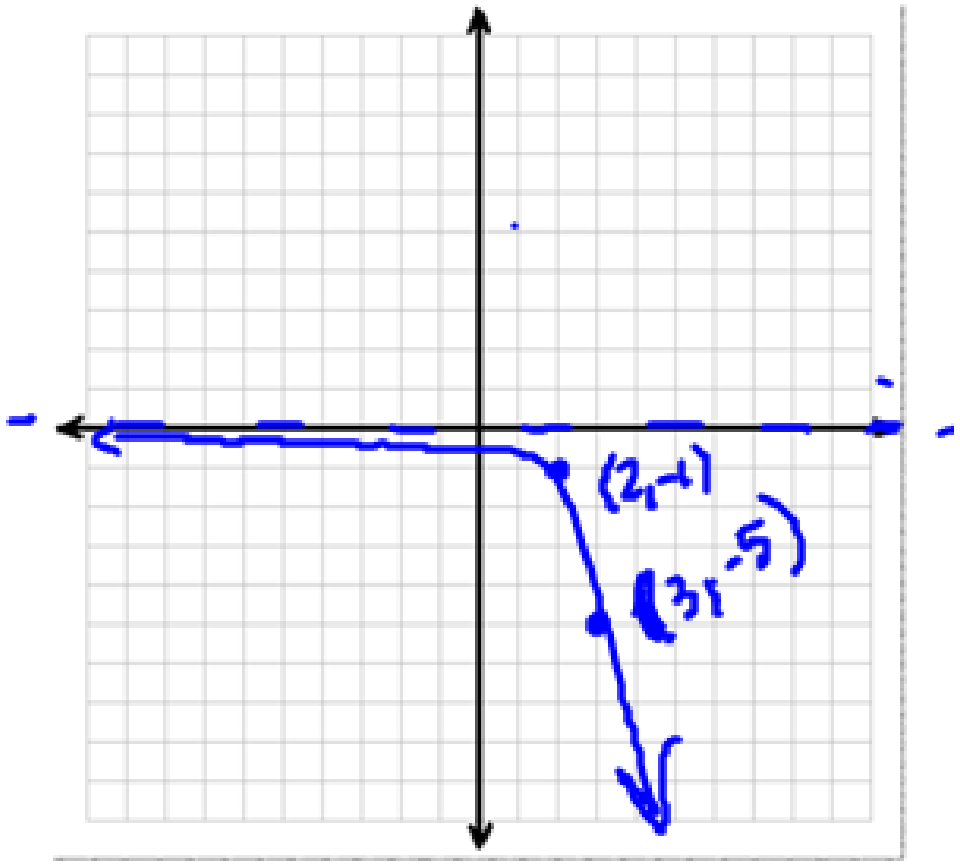
Domain: $\mathbb{R}$

Range: $y<0$

SP

$(0,1) \xrightarrow{h=2} (2,1) \xrightarrow{\text{flip } (-)y} (2,-1)$

$(1,5) \rightarrow (3,5) \rightarrow (3,-5)$



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Example 3:

flips over y-axis → (-1)x

$$y = 2^{-x}$$

h = 0 k = 0

HA: y = 0

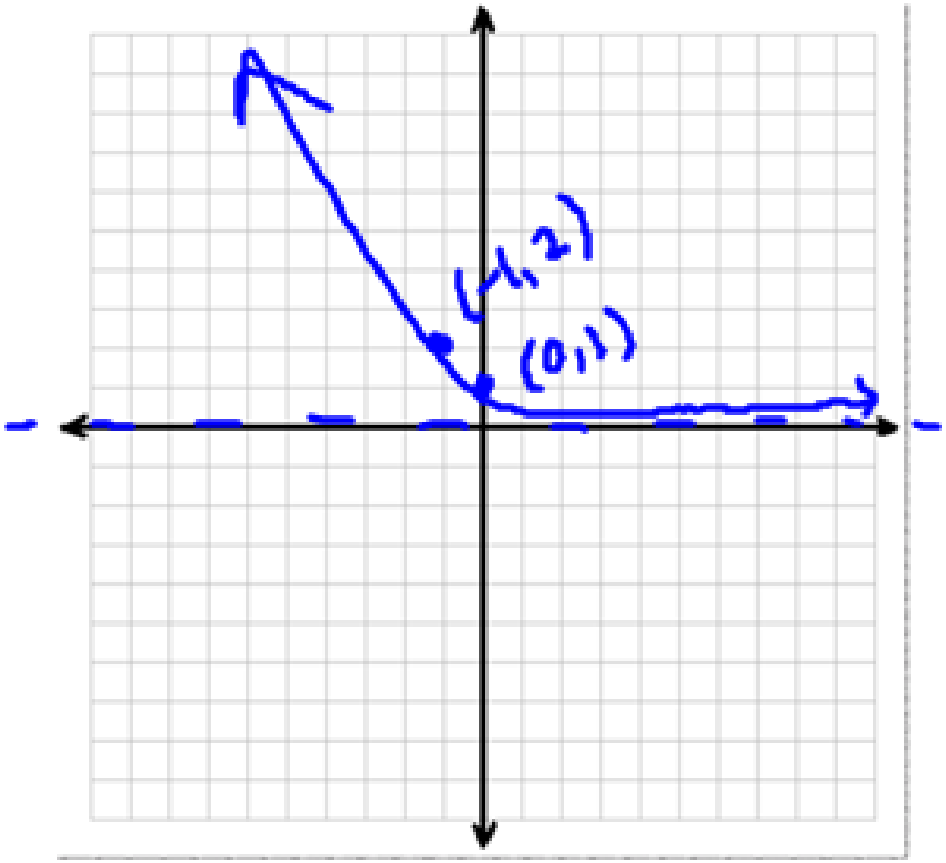
Domain: ℝ

Range: $y > 0$

SP

(0,1) → (0,1) (-1)x

(1,2) → (-1,2)



Example 4:

$$y = -4^{x+3} + 4$$

(-1) y

h = -3 a = -1 k = 4

HA: y = 4

Domain: $\mathbb{R}$

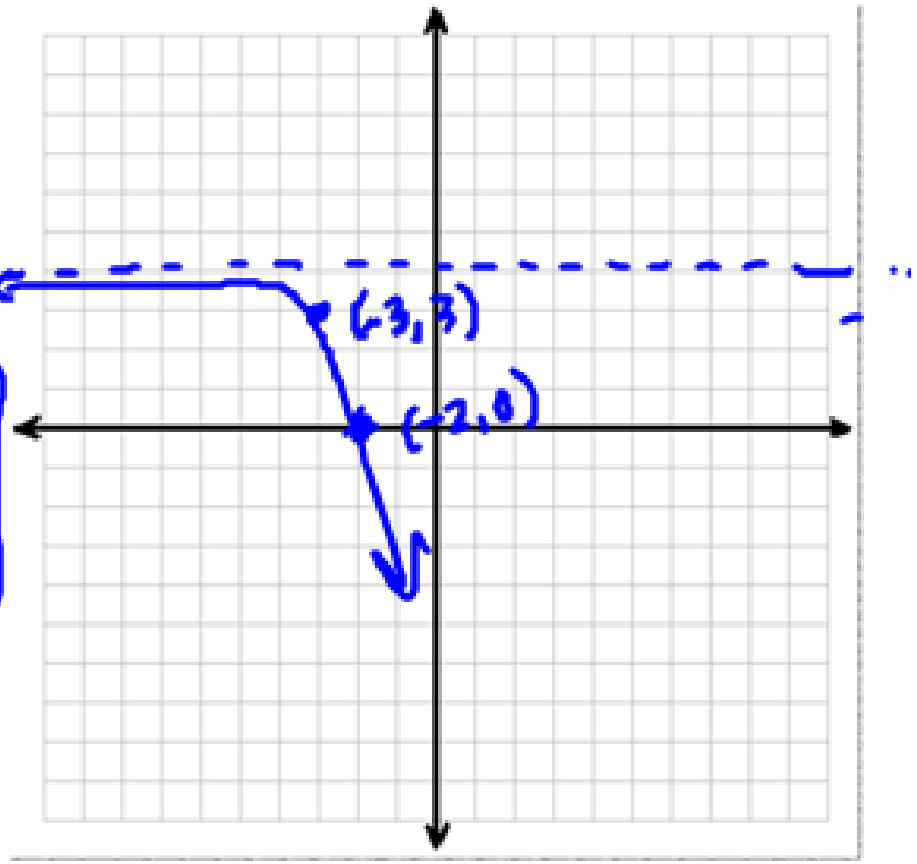
Range: $y < 4$

SP

(0,1) → (-3,1) → (-3,-1) → (-3,3)

(1,4) → (-2,4) → (-2,-4) → (-2,0)

h = -3 ×(-1) k = 4



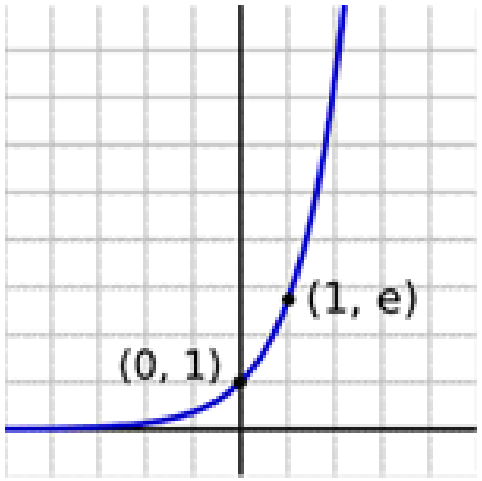
Lesson 43: Exponential Functions

The letter "e" is used to represent a special irrational constant:

$$e \approx 2.71828$$

This number is often used as a base for exponential functions. (We will learn more about "e" in future lessons.)

Graph of $y = e^x$



Example 5:

$h=3 \quad k=2$

$$g(x) = e^{x-3} + 2$$

HA: $y = 2$

Domain:

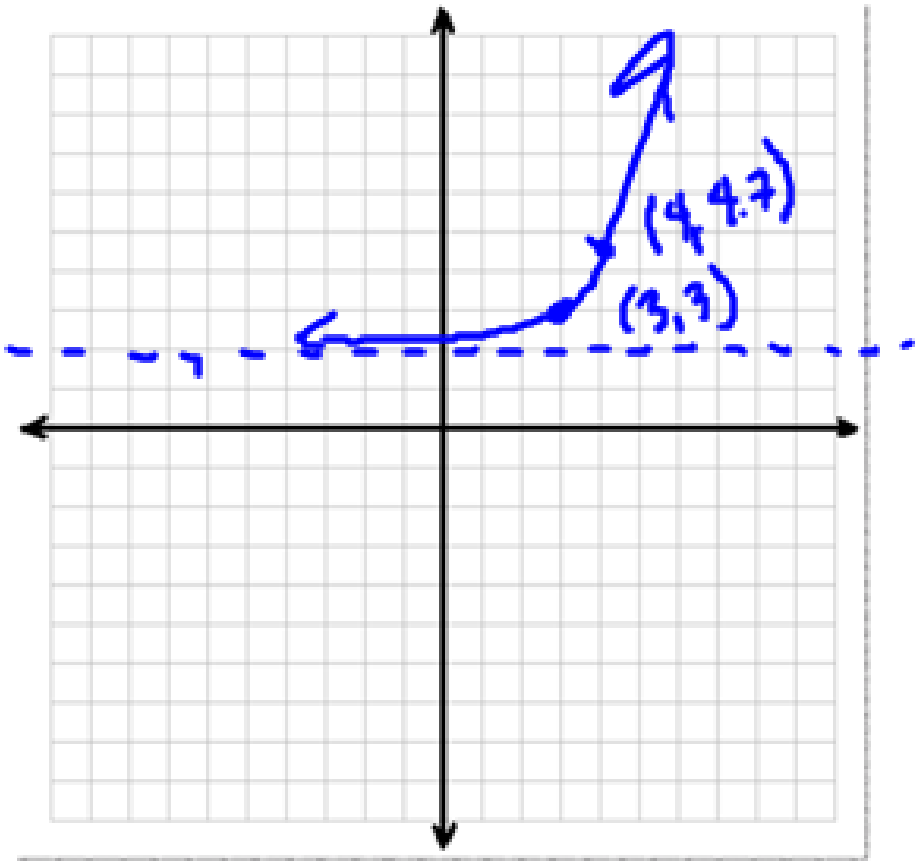
Range:

SP $\quad h=3 \quad k=2$

$(0,1) \rightarrow (3,1) \rightarrow (3,3)$

$(1,e) \rightarrow (4,e) \rightarrow (4,e+2)$

$(1, 2.7)$ $\qquad$ $(4, 4.7)$



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Examples: Evaluate to 3 decimal places.

a.) $e^{5.1} = e^{\wedge(5.1)} \approx 164.022$

b.) $e^{-1.2} = e^{\wedge(-1.2)} = .301$

c.) $e^{\frac{1}{3}} = e^{\wedge(1/3)} = 1.396$

Lesson 43: Exponential Functions

Real life situations involving exponential growth or decay can be modeled using the equation:

Exponential Growth:

$$y = Pe^{rt}$$

where y is final amount, P is initial amount (start) (Principal), r is the growth rate, and t is time. (years)

5% → (.05)

$$y = Pe^{rt}$$

Examples:

round 2 dec

A. Your parents put $2000 in a college fund when you are born. The account pays 5% interest. How much do you have in the account when you turn 18?

$y = ?$

$P = 2000$

5% $r = .05$

$t = 18$

$y = 2000\, e^{(.05 * 18)}$

$y = 2000 * e^{\wedge(.05 * 18)}$ calc

$y = 4919.206$

$\$ 4919.21$

Lesson 43: Exponential Functions

$$y = Pe^{rt}$$

Examples:

B. You would like to have \$15,000 for college on your 20 th birthday. How much would need to be deposited on your 15th birthday if the account pays 7.5% interest?

$y = 15{,}000$

$P = ?$

7.5 $r = .075$

20-15 $t = 5$

$$15000 = Pe^{(.075 \times 5)}$$

$$e^{(.075 \times 5)} \qquad e^{(.075 \times 5)}$$

$$P = \frac{15000}{e^{(.075 \times 5)}}$$

$$P = \$10{,}309.34$$

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Objectives:

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* Horizontal Shift
* Vertical Shift
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Can you?

Homework:

Assignment 43