Lesson 44: Logarithms and Log Graphs

By the end of the lesson, we will be able to:

~ Write Logarithmic Expressions in Exponential form and vice versa.

~ Evaluate expressions in Log form.

~ Graph Log Functions.

Logarithms are the inverse of exponential functions. The expression $\log_b x$ is read "log base b of x". b is the base of the log, and it has the restrictions: b > 0 and b is not equal to 1.

$(b \geq 2)$

$$y = \log_b x \quad \text{means} \quad b^y = x$$

Logs can have any base, but frequently used bases are 10 and e. Common Log has base 10. If the log has no specified base, then it is assumed to be a common log, i.e. $\log x = \log_{10} x$. Natural Log has base e. The notation $\ln$ is used for natural log, i.e. $\ln x = \log_e x$.

$$\begin{cases} y = \log x \text{ means } 10^y = x \\ \\ y = \ln x \text{ means } e^y = x \end{cases}$$

Just like 3 divided by 4 can be writen as $3 \div 4$ OR it can be written with multiplication as $3 \times \frac{1}{4}$, Logs can be rewritten as exponents.

Log Story:

We have a hill with a tree on it. A branch breaks off (the LOG) and rolls down the hill. It squishes the house (b) and knocks the weather vane (y) across the fence (=). The weather vane scares the sheep (x) on the other side of the fence (=) and so the sheep (x) jumps over the fence (=).

$$b^y = x \quad then \quad \log_b x = y$$

Lesson 44: Logarithms and Log Graphs

barn

weathervane

fence (=)

$\log$

$b^y |=| x$

Scared the sheep

$$\log_b x = y \rightarrow b^y = x$$

$$b^y = x \quad then \quad \log_b x = y$$

Examples: Write the log equations in exponential form.

A. $\log_5 125 = 3$

$$5^3 = 125$$

B. $\log_6 \left(\frac{1}{36}\right) = -2$

$$6^{-2} = \frac{1}{36}$$

C. $\ln 5 = 1.609$

$$e^{1.609} = 5$$

D. $\log 10{,}000 = 4$

$$10^4 = 10{,}000$$

Lesson 44: Logarithms and Log Graphs

Examples: Write the exponential equations in log form.

E. $7^3 = 343$

$$\log_7(343) = 3$$

F. $3^{-4} = \frac{1}{81}$

$$\log_3\left(\frac{1}{81}\right) = -4$$

G. $e^3 = 20.086$

$$\log_e(20.086) = 3$$

$$\boxed{\ln(20.086) = 3}$$

H. $10^6 = 1,000,000$

$$\log_{10}(1,000,000) = 6$$

$$\boxed{\log(1,000,000) = 6}$$

Calculators can only evaluate logarithms with base 10 or e. For logarithms with bases other than 10 and e, you can evaluate mentally by asking the question, "The base to what power equals the argument?" ex: $\ln e = 1$

$$e^1 = e$$

L. $\log_6 216 = \underline{3}$ Ask : 6 to what power = 216?

$$6^3 = 216$$

M. $\log_2 \left(\frac{1}{8}\right) = \underline{-3}$ Ask 2 to what power = 1/8?

$$2^{-3} = \frac{1}{8}$$

N. $\log_{25} 5 = \underline{\frac{1}{2}}$ Ask 25 to what power = 5?

$$25^{\frac{1}{2}} = 5^1$$

O. Think about it...

$\log_2 2 = 1$ $\log_2 1 = 0$ $\log_2 0 = \cancel{\emptyset}$ $\log_2 (-1) = \cancel{\emptyset}$

(2 to what power equals...)   no sol.   no sol.

Examples: Evaluate to 3 decimal places. (Calculator)

$$\log(217) =$$

a.) $\log_{10} 217 = 2.336$

b.) $\log \dfrac{2}{3} = \log(2/3) = -.176$

c.) $\ln 23 = \ln(23) = 3.135$

Graphing

Logarithmic and Exponential Functions are inverse functions. This means:

* Logarithmic and exponential graphs are reflections of each other across the line $y = x$.
* Log graphs have a vertical asymptote (instead of horizontal.)
* The special points for a logarithmic graph is $(1,0)$, $(b, 1)$.
* Range and Domain are reversed, so the range of a log graph is $\mathcal{R}$ and the domian is restricted by the asymptote.

Range
$\mathcal{R}$

Lesson 44: Logarithms and Log Graphs

The general equation for a log is $y = \log_b(x - h) + k.$

Parent Graph for $y = \log_b x$:

VA
Vertical asymptote: $\underline{x = h}$ (h tells you where)

The Special Point is $\underline{(1,0)}$ (normally at h+1,0)

The next step is $\underline{(b,1)}$ (normally at b,1)

Other points: $(b^2, 2), (b^3, 3), \ldots$

$y = \log_2 x$
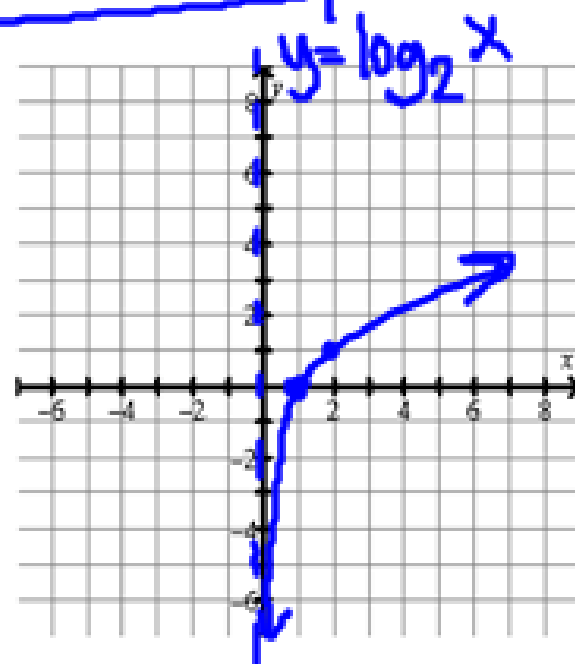
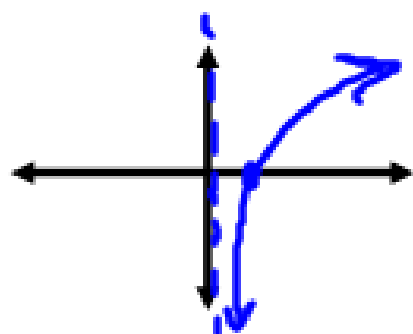## Translations:

h affects horizontal shift
k affects vertical shift

Reflections:

$y = \log_b x$
Normal parent graph
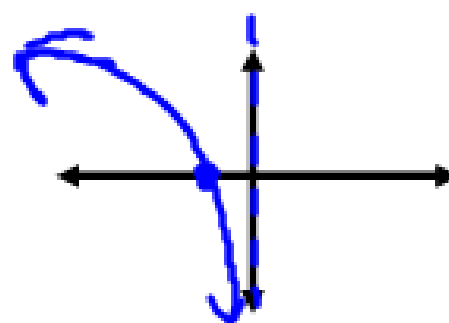Direction:

$y = \log_b(-x)$
Flips the graph ___y-axis___
Direction: **Mult x's by -1**
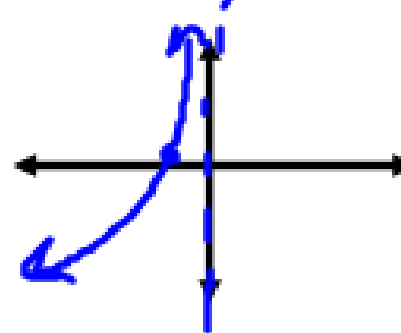

$y = -\log_b x$
Flips the graph ___X-axis___
Direction: **Mult y's by -1**

$y = -\log_b(-x)$
Flips the graph ___x+y-axis___
Direction: **; Mult x+y's by -1**

Steps to Graph a Logarithm Function: $x = h$

1. Identify and graph the **Vertical asymptote (VA)**.

2. Write down special points (1, 0) (b, 1).

3. Add the "h" value to the X's in your special points.

4. If "a" is negative (outside parentheses), make the "y" value in the special points negative.
   If "x" is negative (inside parentheses), make the "x" value in the special points negative.

   ~~ **This is our reflection step.** ~~

5. Add the "k" value to the Y's in your special points.

6. Plot the points and connect the dots.

   (Remember arrows!)

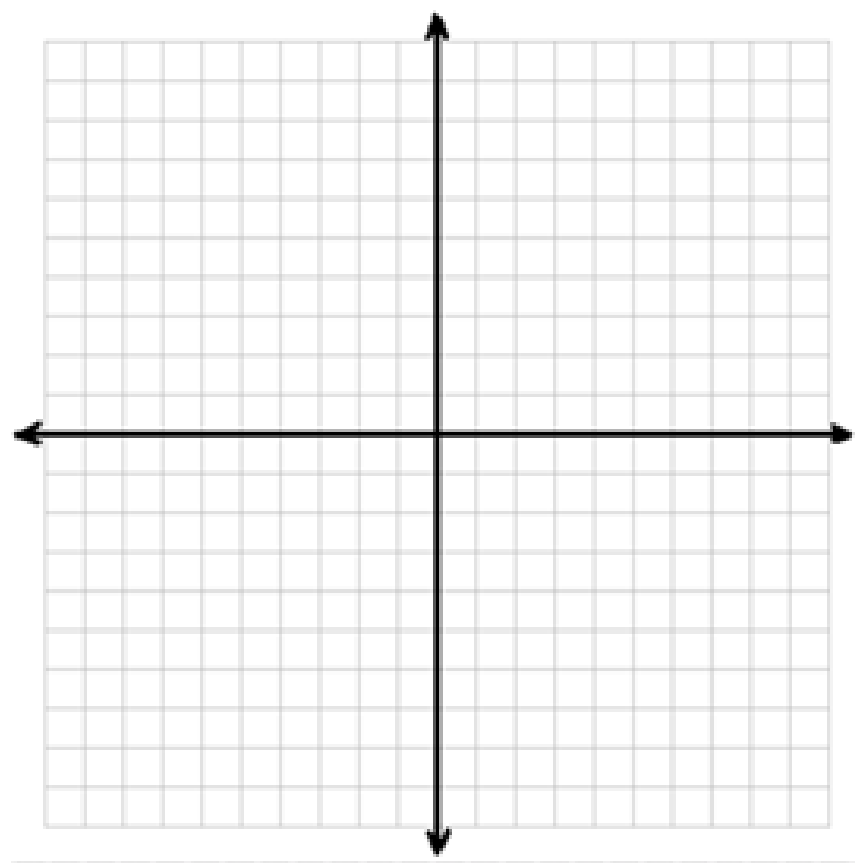   The graph will always go vertically along the asymptote.

Example 1: Graph each log function

$$y = \log_5 x$$

VA:_____

Domain:

Range:

Lesson 44: Logarithms and Log Graphs

Example 2: Graph

$$f(x) = \log_3(x + 2)$$

$b = 3$

$h = -2$ $k = 0$

VA: $x = -2$

Domain: $x > 2$

Range: $\mathbb{R}$

SP

$(1, 0) \rightarrow (-1, 0)$

$(3, 1) \rightarrow (1, 1)$
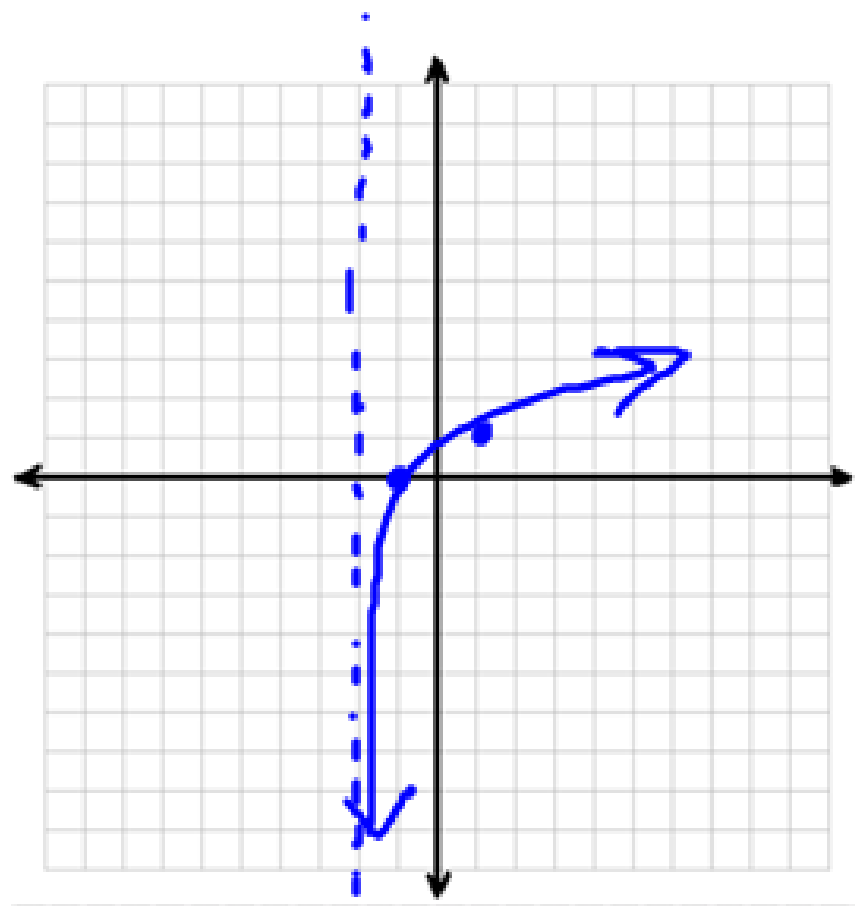
Example 3: Graph

$$y = \log_3(x) + 2$$

$b = 3$

$h = 0 \quad \nearrow \quad k = 2$

VA: _____ $x = 0$ _____

Domain: $x > 0$

Range: $\mathbb{R}$

$\underline{SP}$

$(1, 0) \xrightarrow{k=2} (1, 2)$

$(3, 1) \rightarrow (3, 3)$
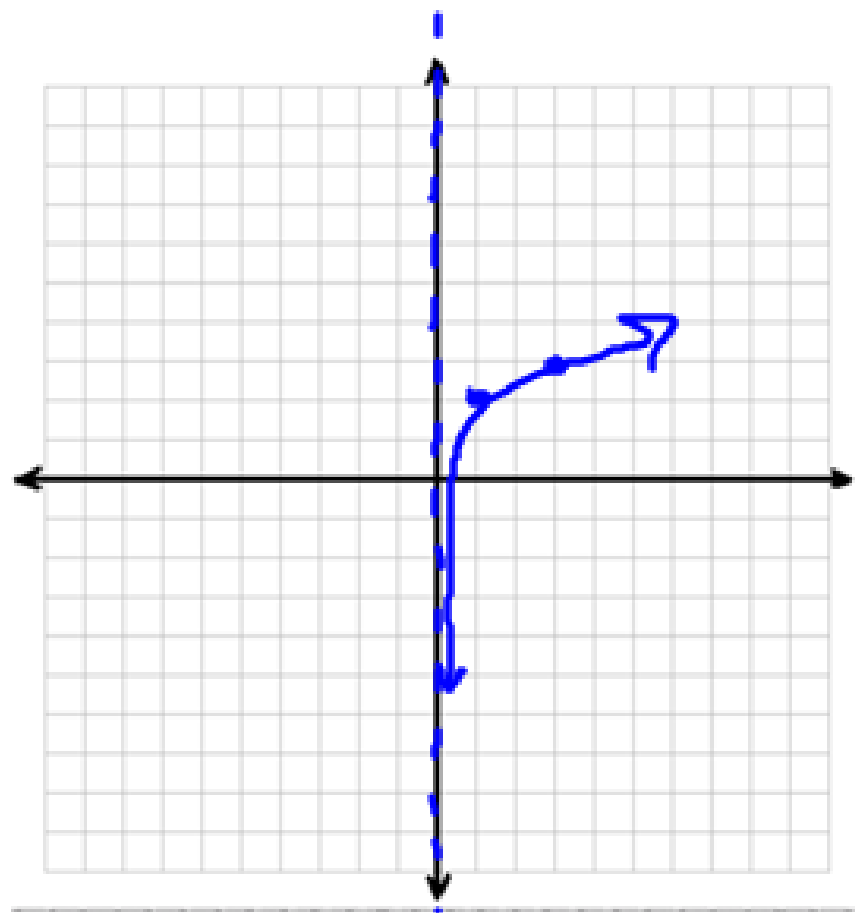
Lesson 44: Logarithms and Log Graphs

Example 4: Graph

$$y = \log_4(-x) + 3$$

$b = 4$

mult x's by -1

$h = 0$ ← $k = 3$

VA: $X = 0$

Domain: $x < 0$

Range: $\mathbb{R}$

SP

$\underline{}$ x(-1)

$(1,0) \xrightarrow{} (-1,0) \xrightarrow{k=3} (4,3)$

$(4,1) \xrightarrow{} (-4,1) \xrightarrow{} (-4,4)$
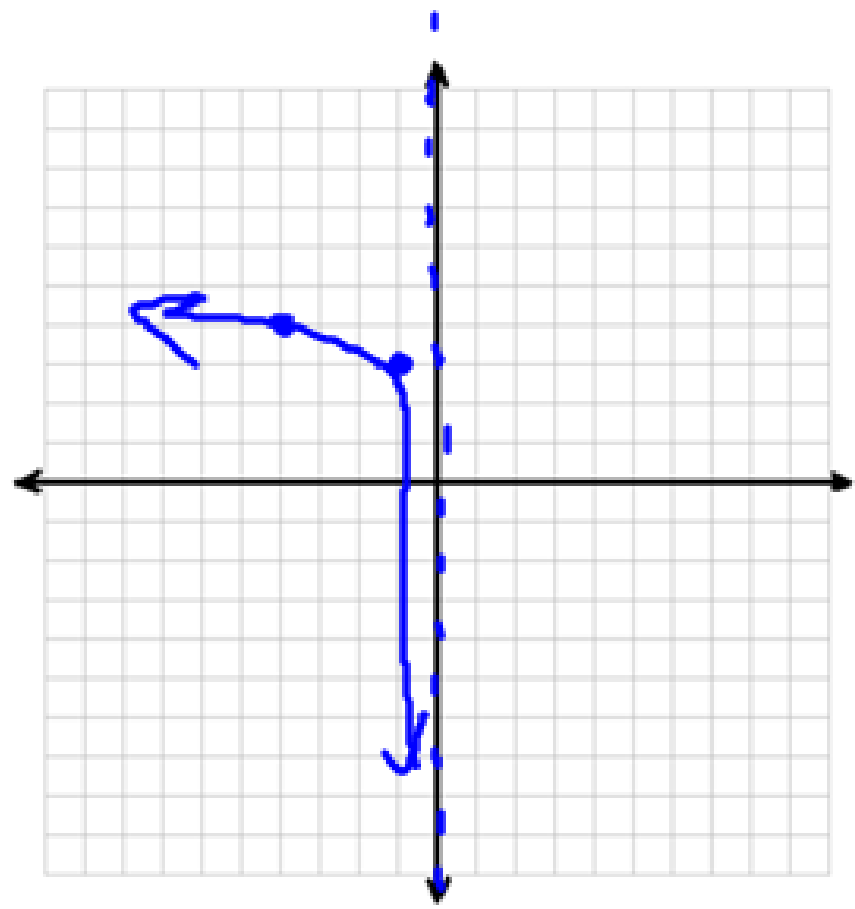
$$b = 5$$

Example 5: Graph

$$y = -\log_5(x + 1) + 2 \qquad h = -1 \qquad a = -1 \qquad k = 2$$

mult y's by -1
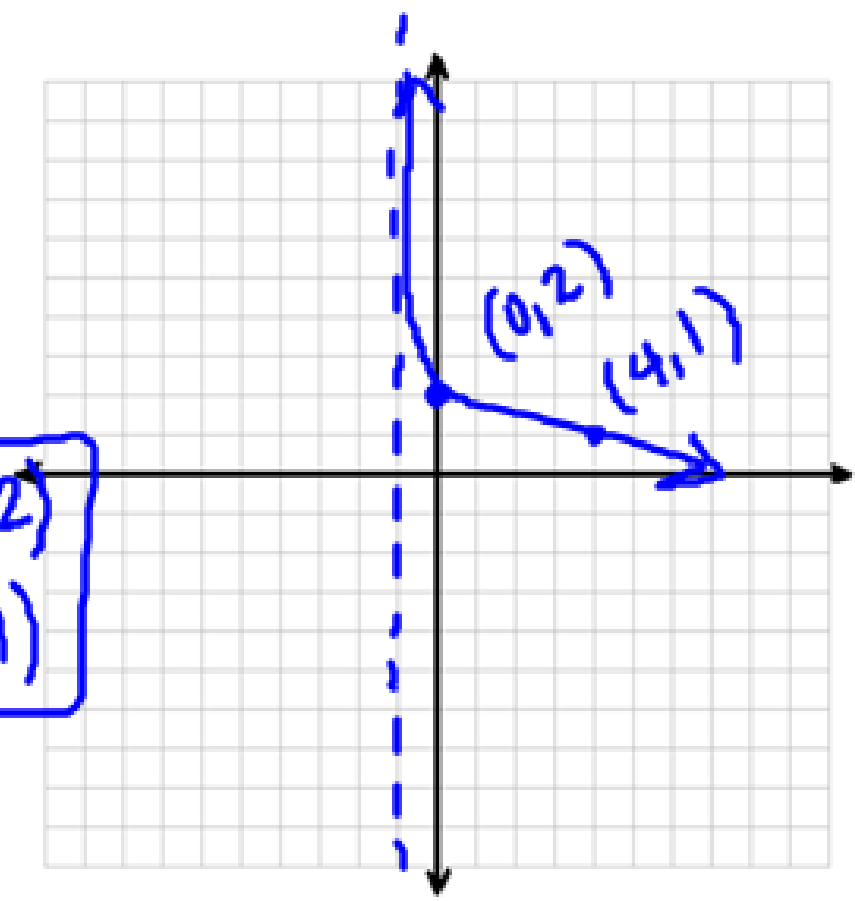
VA: $\underline{\qquad x = -1 \qquad}$

Domain: $x > -1$

Range: $\mathbb{R}$

SP

$$(1, 0) \xrightarrow{h = -1} (0, 0) \xrightarrow{(1)y} (0, 0) \xrightarrow{k = 2} \boxed{(0, 2)}$$

$$(5, 1) \rightarrow (4, 1) \rightarrow (4, -1) \rightarrow \boxed{(4, 1)}$$

(0, 2)

(4, 1)

Lesson 44: Logarithms and Log Graphs

Example 6: Graph

$$y = \ln(x-1) + 1$$

$b = e \approx 2.7$

$h = 1$ $\cancel{x=}$ $k = 1$

VA: $\underline{\quad x = 1 \quad}$

Domain: $x > 1$

Range: $\mathbb{R}$

$\dfrac{SP}{}$

$(1, 0) \xrightarrow{h=1} (2, 0) \xrightarrow{k=1} \boxed{\begin{array}{c}(2, 1)\end{array}}$

$(e, 1) \rightarrow (e+1, 1) \rightarrow \boxed{\begin{array}{c}(e+1, 2)\end{array}}$

$\quad\quad\quad\quad\text{or}\quad\quad\quad\quad\text{or}$

$(2.7, 1) \quad\quad (3.7, 1) \quad\quad (3.7, 2)$
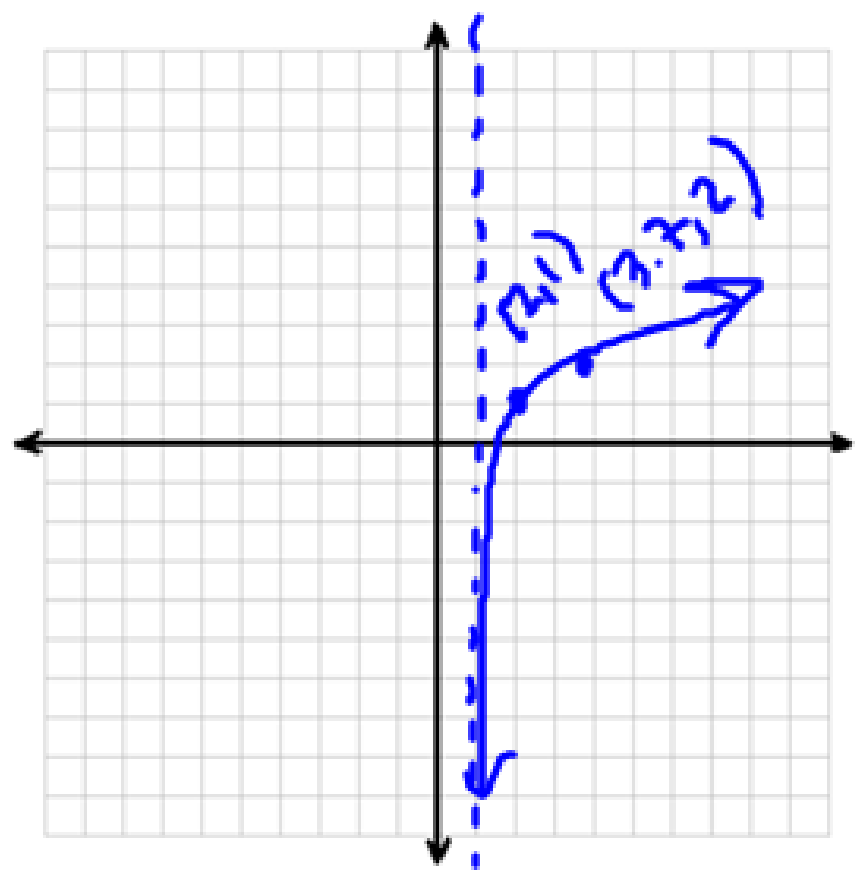
Our objective today was to:

~   Write Logarithmic Expressions in
        Exponential form and vice versa.
~   Evaluate expressions in Log form.
~   Graph Log Functions.

Can you do this?

Homework:

Assignment 44
& Log Monster

You can get the log monster from the website below if you lose the one from class.
http://cunninghammath.weebly.com/assignments---algebra-2.html
Look under Term 3. :)