# Lesson 45: Log Rules

By the end of the lesson, we will be able to:

- Understand Properties of Logarithms
  - Change the base of log functions so we can evaluate them.
  - Expand Logarithmic Expressions.
  - Condense Logarithmic Expressions.

## Lesson 45: Log Rules

There are 4 properties of logarithms that are used to evaluate and rewrite log expressions.

The first Property of Logs is:

**Change of Base Property:** $\log_b x = \frac{\log x}{\log b}$ or $\log_b x = \frac{\ln x}{\ln b}$

**Lesson 45: Log Rules**

round to 3 dec.

Evaluate using a calculator and the change of base property:

**Change of Base Property:** $\log_b x = \frac{\log x}{\log b}$ or $\log_b x = \frac{\ln x}{\ln b}$

A. $\log_4 12$ (trick: b is for bottom.... it goes on bottom.)

$\frac{\log(12)}{\log(4)}$ or $\frac{\ln(12)}{\ln(4)} = \boxed{1.792}$

B. $\log_{20} 26.3$

$\frac{\log(26.3)}{\log(20)}$ or $\frac{\ln(26.3)}{\ln(20)} = \boxed{1.091}$

C. $\log_5 125$

$\frac{\ln(125)}{\ln(5)} = \boxed{3}$

# Two more important Properties of Logs

**Product Property:** $\log_b(mn) = \log_b m + \log_b n$

**Quotient Property:** $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

## Lesson 45: Log Rules

### Expand each log expression

D. $\log_2(5xy)$

$= \log_2(5) + \log_2(x) + \log_2(y)$

E. $\ln\left(\frac{a}{(b+1)}\right)$

$= \ln(a) - \ln(b+1)$

F. $\log_3\left(\frac{pq}{6}\right)$

$= \log_3(p) + \log_3(q) - \log_3(6)$

G. $\ln\left(\frac{w}{xy}\right)$

$= \ln(w) - (\ln(x) + \ln y)$

$\ln(w) - \ln(x) - \ln(y)$

Numbers on top are added

Numbers on bottom are subtracted

**Product Property:** $\log_b(mn) = \log_b m + \log_b n$

**Quotient Property:** $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

## Lesson 45: Log Rules

Condense each log expression (- goes on bottom, + goes on top)

H. $\ln a + \ln b + \ln c$

$= \ln(abc)$

I. $\log 5 - \log x - \log(y-4)$

$= \log\left(\frac{5}{x(y-4)}\right)$

J. $\log_3 5 - \log_3 u + \log_3 6$

$= \log_3\left(\frac{5 \cdot 6}{u}\right) = \log_3\left(\frac{30}{u}\right)$

## Lesson 45: Log Rules

# The fourth Property of Logs is:

**Power Property:** $\log_b(m^p) = p \cdot \log_b m$

(Follow the previos rules and the power goes out front)

Expand each log expression using the Properties of Logs.

K. $\log_5\left(\frac{x^3}{y^2}\right)$

$= \log_5(x^3) - \log_5(y^2)$

$= 3\log_5(x) - 2\log_5(y)$

L. $\ln(a^5\sqrt{b})$

$= \ln(a^5) + \ln(\sqrt{b})$

$= \ln(a^5) + \ln(b^{\frac{1}{2}})$

$= 5\ln(a) + \frac{1}{2}\ln(b)$

## Lesson 45: Log Rules

**Power Property:** $\log_b(m^p) = p \cdot \log_b m$

Remember, fractions rewrite as roots ( $x^{\frac{1}{2}} = \sqrt[2]{x}$ )

# Practice - Condense each log expression

M. $2\ln x + \frac{1}{2}\ln(z+2)$

$= \ln x^2 + \ln (z+2)^{\frac{1}{2}}$

$= \ln x^2 + \ln \sqrt{z+2}$

$= \boxed{\ln\left(x^2\sqrt{z+2}\right)}$

N. $2\log 5 + \frac{1}{3}\log u - 4\log 3$

$= \log(5^2) + \log(u^{\frac{1}{3}}) - \log(3^4)$

$= \log(25) + \log\sqrt[3]{u} - \log(81)$

$= \boxed{\log\left(\frac{25\sqrt[3]{u}}{81}\right)}$

# LOG RULES

## CHANGE OF BASE:

$$\log_b x = \frac{\log x}{\log b} \text{ or } \frac{\ln x}{\ln b}$$

$$\log_b (x)^n = n \cdot \log_b (x)$$

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

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CAN YOU???

Homework:

Assignment 45