Lesson 46: Solving Exponential Equations

By the end of this lesson you will be able to:

~ Solve exponential equations using common bases

~ Solve exponential equations using logarithms

Solving Exponential Equations

If bases are equal, then the exponents can be set equal. This is one way to solve exponential equations.

Property of equality for exponential functions: If $b^x = b^y$, then $x = y$

For example: If $2^x = 2^4$ then, $x = 4$

$2^{x-1} = 2^4 \rightarrow x - 1 = 4$

$+1 \quad +1$

$\boxed{x=5}$

Solving Using Common Bases

Steps :

1. Find a common base for both sides of the equation.
2. Rewrite each base as a power of the common base.
3. Simplify the exponent expression if necessary. (Remember: power to a power is multiplied.) $(2^3)^2 = 2^6$
4. Set the exponents equal to each other to make a new equation.
5. Solve the new equation

Example: Solve for the variable $81 = 27^{m-1}$

Step 1: The common base for 81 and 27 is base $\underline{3}$.

$\underline{3}^4 = 81 \qquad \underline{3}^3 = 27$

Step 2: Rewrite each base as a power of the common base.

$3^4 = (3^3)^{(m-1)}$

Step 3: Simplify the exponent expression.

$3^4 = 3^{3m-3}$

Step 4: Set exponents equal to each other.

$$\begin{aligned} 4 &= 3m - 3 \\ +3 & \quad\;\; +3 \end{aligned}$$

Step 5: Solve the new equation.

$\frac{7}{3} = \frac{3m}{3} \rightarrow \boxed{m = 7/3}$

Example: Solve for the variable $2^{3n-9} = 64$

Step 1: The common base for 2 and 64 is base 2.

$2^6 = 64$

Step 2: Rewrite each base as a power of the common base.

$2^{3n-9} = 2^6$

Step 3: Simplify the exponent expression.

Step 4: Set exponents equal to each other.

$$\begin{aligned} 3n - 9 &= 6 \\ +9 \quad &\;\; +9 \end{aligned} \rightarrow \frac{3n}{3} = \frac{15}{3} \rightarrow \boxed{n = 5}$$

Step 5: Solve the new equation.

Example: Solve for the variable

$$16^{2n+1} = \frac{1}{32}$$

$2^4 = 16 \qquad 2^{-5} = \frac{1}{32}$

$(2^4)^{(2n+1)} = 2^{-5}$

$2^{8n+4} = 2^{-5}$

$$\begin{aligned} 8n + 4 &= -5 \\ -4 \quad & \;\; -4 \\ \hline 8n &= -9 \end{aligned}$$

$\frac{8n}{8} = \frac{-9}{8}$

$\boxed{n = -9/8}$

Solving Using Logarithms

Steps

1. Isolate the exponential expression, if necessary.
2. Take the ln of both sides.
3. Move the exponent out in front of the log expression. (Power property of logarithms.)
4. Divide both sides of the equation by the log. (Except when using Ln *e*.)
5. Continue to solve for *x*, if necessary.
6. Write answers in exact form (with logs), and also as a decimal approximation (round to 3 places).

Example: Solve for the variable $8^{x+4} - 4 = 100$

$$\begin{array}{r} 8^{x+4} - 4 = 100 \\ +4 \quad\quad +4 \\ \hline \end{array}$$

Step 1: Isolate the exponential expression.

$$8^{x+4} = 104$$

Step 2: Take the ln of both sides.

$$\ln(8^{x+4}) = \ln(104)$$

Step 3: Move the exponent out in front of the log expression.

$$\frac{(x+4)\ln(8)}{\ln(8)} = \frac{\ln(104)}{\ln(8)}$$

Step 4: Divide both sides of the equation by the log.

$$x+4 = \frac{\ln(104)}{\ln(8)}$$
$$-4 \quad\quad -4$$

Step 5: Continue to solve for x.

$$\boxed{x = \frac{\ln(104)}{\ln(8)} - 4}$$

calc: $\frac{\ln(104)}{\ln(8)}$ "enter" ans−4 "enter"

Step 6: Find the decimal value.

$$\boxed{x = -1.767}$$

Example: Solve for the variable $7^{2x} = 56$

Step 1: Isolate the exponential expression.

Step 2: Take the ~~log~~ ln of both sides.

$$\ln(7^{2x}) = \ln(56)$$

Step 3: Move the exponent out in front of the log expression.

$$\frac{\cancel{2}x\,\cancel{\ln(7)}}{\cancel{2\ln(7)}} = \frac{\ln(56)}{2\ln(7)}$$

Step 4: Divide both sides of the equation by the log.

$$\boxed{x = \frac{\ln(56)}{2\ln(7)}}$$

Step 5: Continue to solve for x.

Calc: $\dfrac{\ln(56)}{(2 * \ln(7))}$ "enter"

Step 6: Find the decimal value.

$$\boxed{x = 1.034}$$

Example: Solve for the variable

$$3e^x - 5 = 22$$

$$+5 \quad +5$$

$$\frac{3e^x}{3} = \frac{27}{3}$$

$$e^x = 9$$

$$\ln(e^x) = \ln(9)$$

$$x \cdot \ln(e) = \ln(9)$$

$$\boxed{x = \ln(9)} \quad \boxed{x = 2.197}$$

$$\ln(e) = \log_e(e) = 1$$

$$\ln(e) = 1$$

Example: Solve for the variable

$$6^x = 42$$

$$\boxed{x = \frac{\ln(42)}{\ln(6)} = 2.086}$$

Lesson 48: Exponential Growth and Decay

Real life situations involving exponential growth or decay can be modeled using the equation:

Exponential Growth and Decay:

$$\boxed{y = Pe^{rt}}$$

where y is final amount, P is initial amount (Principal), r is the growth rate, and t is time.

Examples of exponential growth include:

~Amount of money in an account with interest

~Population growth

~Appreciation or depreciation of property values

~Radioactive decay of elements.

$$y = Pe^{rt}$$

Examples:

A. Your parents put $2000 in a college fund when you are born. The account pays 5% interest. How much do you have in the account when you turn 18?

$y =$

$P =$

$r =$

$t =$

When trying to find growth rate or time (r or t), logarithms must be used to solve the equation.

Since the equations use e, it is easiest to use natural log (Ln) to solve.

$$y = Pe^{rt}$$

Examples:

E. If a city population is about 112,000 people now, and if the population grows continuously at an annual rate of 4%, How long will it take to reach 250,000 people?

$y = 250{,}000$

$P = 112{,}000$

$r = .04$

$t = ?$

$$\frac{250{,}000}{112{,}000} = \frac{112{,}000e^{.04t}}{112{,}000}$$

$$\frac{250{,}000}{112{,}000} = e^{.04t}$$

$$\ln\left(\frac{250}{112}\right) = \ln e^{.04t}$$

$$\frac{\ln\left(\frac{250}{112}\right)}{.04} = \frac{.04t}{.04}$$

$$\boxed{t = 20.07 \text{ yrs}}$$

$$y = Pe^{rt}$$

Examples:

F. The Diaz family bought a new house 20 years ago for $80,000. The house is now worth $250,000. Assuming a steady rate of growth, what was the yearly rate of appreciation?

$y = 250{,}000$

$P = 80{,}000$

$r = ?$

$t = 20$

$$\boxed{r = 5.70\%}$$

$$\frac{250{,}000}{80{,}000} = \frac{80{,}000\, e^{20r}}{80{,}000}$$

$$\frac{250}{80} = e^{20r}$$

$$\frac{\ln\left(\frac{250}{80}\right)}{20} = \frac{20r}{20} \cancel{\ln e}$$

$$r = .0570$$

$$\boxed{y = Pe^{rt}}$$

Examples:

G. How long will it take a sum of money to double if the interest rate is 4%?

$y = 2x$

$P = x$

$r = .04$

$t = ?$

$$\frac{2\cancel{x}}{\cancel{x}} = \frac{xe^{.04t}}{x}$$

$$2 = e^{.04t}$$

$$\ln(2) = \ln e^{.04t}$$

$$\frac{\ln(2)}{.04} = \frac{.04t}{.04}$$

$$\boxed{t = 17.33\text{yrs}}$$

Can you?

~ Solve exponential equations using common bases

~ Solve exponential equations using logarithms

~Homework~

Journal 46

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Assignment ~~46~~

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