By the end of the lesson, we will be able to:

~ Solve log equations

~ Solve equations with more than one log

Lesson 47: Solving Log Equations

One way to solve log equations is to rewrite in exponential form. We use this method when there is log expression on one side of the equation, and a number on the other side.

Converting between exponential and log form:

$$\boxed{\text{If } \log_b x = y, \quad \text{then } b^y = x}$$

Examples: Convert to exponent form and solve.

a.) $\log_6 x = 2$

$6^2 = x$

$\boxed{36 = x}$

b.) $\log_4(3x) = 3$

$\frac{4^3}{3} = \frac{3x}{3}$

$x = \frac{4^3}{3}$

$\boxed{x = \frac{64}{3}}$ or $21\frac{1}{3}$

Examples: Convert to exponent form and solve.

c.) $5 + 2\ln x = 9$

$-5 \qquad\qquad -5$

$\frac{2 \cdot \ln(x)}{2} = \frac{4}{2}$

$\ln(x) = 2$

$\boxed{e^2 = x}$

Lesson 47: Solving Log Equations

If there is more than one log expression in the equation, the logs must first be condensed.

Example: $\log_2 x + 4\log_2 3 = 4$

$\log_2(x) + \log_2(3^4) = 4$

$\log_2(x) + \log_2(81) = 4$

$\log_2(81x) = 4$

$2^4 = 81x$

$16 = 81x$

$\frac{16}{81} = \frac{81x}{81}$

$\boxed{x = \frac{16}{81}}$

Lesson 47: Solving Log Equations

Important!!!

Always Check Your Solutions!

Some solutions may be extraneous.

A solution is **extraneous** if...

Lesson 47: Solving Log Equations

A solution is **extraneous** if...

it is a solution of the simplified form of an equation that does not satisfy the original equation.

inside of log cannot be zero or negative

"argument"

"Sheep of log" → $\log_2(\_)$

Examples: Condense and solve.

a.) $\log(x+2)+\log(x-1)=1$

$$\log\big((x+2)(x-1)\big)=1$$

$$\log(x^2-x+2x-2)=1$$

$$\log_{10}(x^2+x-2)=1$$

$$10^1=x^2+x-2$$

$$-10 \qquad\qquad -10$$

$$0=x^2+x-12$$

$$0=(x+4)(x-3)$$

$$x+4=0 \qquad x-3=0$$

$$\boxed{\cancel{x=-4} \qquad x=3}$$

checked

Check!

$$\log(x+2) + \log(x-1) = 1$$

$\cancel{x = -4}$

$\log(\underbrace{-4+2}_{-2}) + \log(-4-1) = 1$

$x = 3$

$\log(3+2) + \log(3-1) = 1$

✓

Examples: Condense and solve.

b.) $\log_5(4x+7) - \log_5 x = 2$

$\log_5\left(\frac{4x+7}{x}\right) = 2$

$5^2 = \frac{4x+7}{x}$

$x \cdot 25 = \frac{4x+7}{\cancel{x}} \cdot \cancel{x}$

$25x = 4x+7$
$-4x \quad -4x$

$\frac{21x}{21} = \frac{7}{21}$

$\boxed{x = \frac{1}{3}}$

Check:

$\log_5\left(4\left(\frac{1}{3}\right)+7\right) - \log_5\left(\frac{1}{3}\right) = 2$

$+ \qquad +$

✓

Another way to solve log equations is to use the following property of equality:

Property of equality for log functions:

$$\text{If } \log_b x = \log_b y, \text{ then } x = y$$

If there is a single log on both sides, with the same base, then the arguments can be set equal.

Examples:

a.) $\log_3(3x - 5) = \log_3(x + 7)$

$$\begin{aligned} 3x - \cancel{5} &= \cancel{x} + 7 \\ -x + \cancel{5} &\quad -\cancel{x} + 5 \\ \hline \frac{2x}{2} &= \frac{12}{2} \end{aligned}$$

$$\boxed{x = 6}$$

Check

$\log_3(3(6)-5) = \log_3(6+7)$

$\log_3(13) = \log_3(13)$ ✓

Examples:

b.) $\log(x^2 - 6) = \log x$

If there is more than one log expression in the equation, the logs must first be condensed.

Examples:

a.) $\log_2(x+3) - \log_2(2x-1) = \log_2 2$

$\log_2\left(\frac{x+3}{2x-1}\right) = \log_2(2)$ → $\boxed{x = \frac{5}{3}}$

$\frac{(2x-1)}{1} \cdot \frac{x+3}{\cancel{2x-1}} = 2(2x-1)$

$x + \cancel{3} = \cancel{4x} - 2$

$-4x - 3 \quad -4x - 3$

$\frac{-3x}{-3} = \frac{-5}{-3}$

Check:

$\log_2\left(\frac{5}{3}+3\right) - \log_2\left(2\left(\frac{5}{3}\right)-1\right) = \log_2(2)$

$+ \qquad +$

✓

Real life situations involving exponential growth or decay can be modeled using the equation:

Exponential Growth and Decay:

$$\boxed{y = Pe^{rt}}$$

start

where y is final amount, P is initial amount (Principal), r is the growth rate, and t is time.

This equation $y = Pe^{rt}$ is used when we are compounding interest continually (always).

But sometimes, we only compound interest annually (once a year). We use this equation when we do that: $y = P(1 + r)^t$

$(1+r)-1 = r$

In this equation $y = P(1+r)^t$, r stands for rate.

~ If r is positive, we are growing.

~ If r is negative, we are decaying (decreasing).

Lesson 48: Exponential Growth and Decay

$$\boxed{y = P(1+r)^t}$$

Examples: Give the percent growth or decay represented by the following equation.

B. $y = 3000(\underline{1.65})^t$

$1.65 - 1 = .65$

$\boxed{65\% \text{ growth}}$

$$y = P(1+r)^t$$

Examples: Give the percent growth or decay represented by the following equation.

c. $y = 4000(.75)^t$

$.75 - 1 = -.25$

$\boxed{25\% \text{ decay}}$

$$y = P(1+r)^t$$

Examples: Give the percent growth or decay represented by the following equation.

D. $y = 3500\left(\frac{3}{4}\right)^t$

$\frac{3}{4} - 1 = .75 - 1 = -.25$

$\boxed{25\% \text{ decay}}$

Another application of exponential equations is **radioactive decay**. Radioactive elements decay over time, and the rate of decay is constant. Instead of a decay rate (r), the decay is often given as the **half life** of the substance. Half life is the amount of time it takes for half of the mass to decay.

When given the half life of an element, the decay rate (r) can be found using the formula:

Decay rate when given half life:

$$\boxed{r = \frac{-\ln 2}{half\ life}}$$

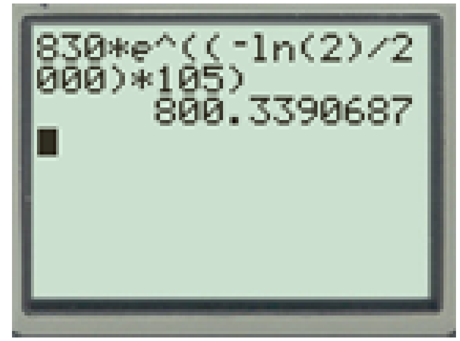
$$y = Pe^{rt}$$

Examples:

2 dec.

Element X has a half-life of 2000 years. A sample starts with 830 grams.

H. How many grams will be left in 105 years?



$y = ?$

$P = 830$

$r = \longrightarrow \dfrac{-\ln(2)}{2000}$

$t = 105$

$$y = 830e^{\left(\frac{-\ln(2)}{2000} * 105\right)}$$

$$\boxed{y = 800.34\text{ g}}$$

$$r = \frac{-\ln 2}{half\ life}$$

2000

Lesson 48: Exponential Growth and Decay

$$\boxed{y = Pe^{rt}}$$

Examples:

Element X has a half-life of 2000 years. A sample starts with 830 grams.

```
ln(20/83)/(-ln(2
)/2000)
         4106.222673
```

I. How long until 200 grams remain?

$y = 200$

$P = 830$

$r = \rightarrow \dfrac{-\ln(2)}{2000}$

$t = ?$

$$\frac{200}{830} = \frac{830e^{\left(\frac{-\ln(2)}{2000} * t\right)}}{830}$$

$$\frac{200}{830} = e^{\frac{-\ln(2)}{2000} * t}$$

$$\frac{\ln\left(\frac{200}{830}\right)}{\left(\frac{-\ln(2)}{2000}\right)} = \frac{\frac{-\ln(2)}{2000} * t}{\frac{-\ln(2)}{2000}}$$

$$t = \frac{\ln\left(\frac{20}{83}\right)}{\left(\frac{-\ln(2)}{2000}\right)}$$

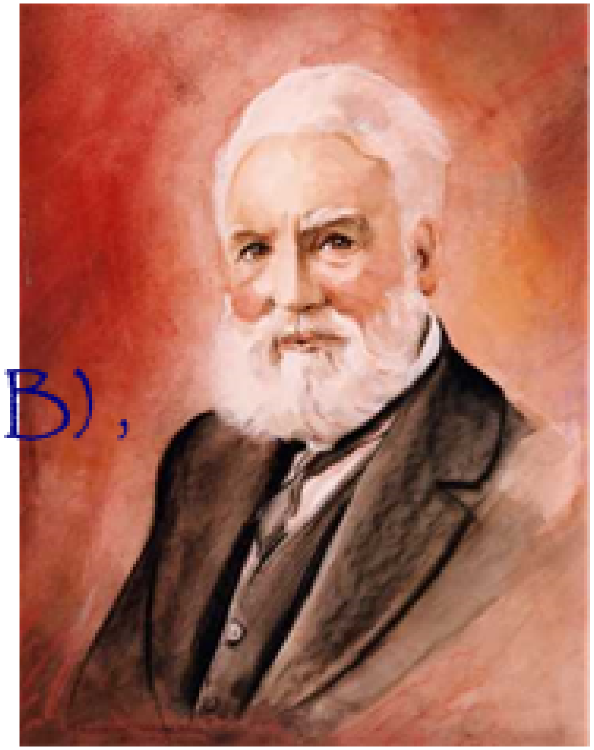
$$\boxed{t = 4106.22 \text{ yrs}}$$

$$r = \frac{-\ln 2}{\text{half life}}$$

2000

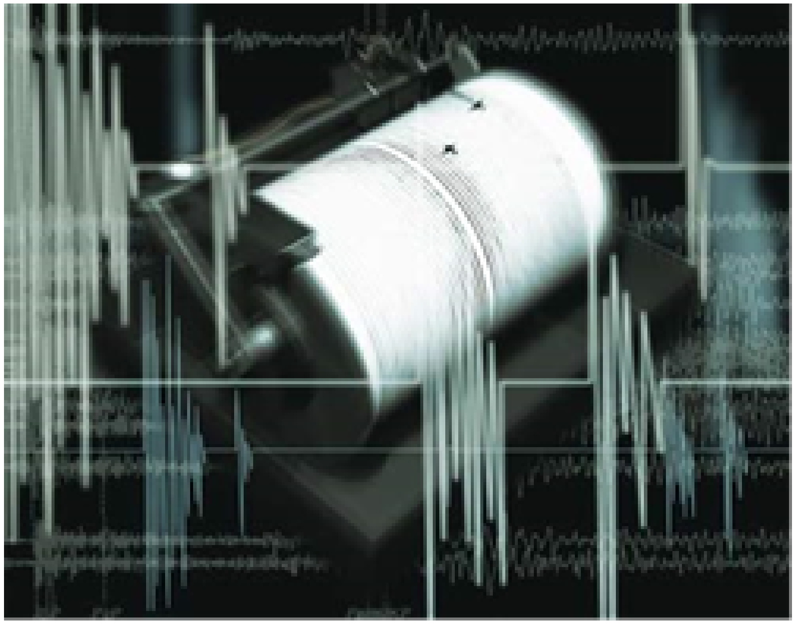
When are logs used?

The bel (symbol B) is a unit of measure which is the base -10 logarithm of ratios, such as power levels and voltage levels. It is mostly used in telecommunication, electronics, and acoustics. The Bel is named after telecommunications pioneer Alexander Graham Bell. The decibel (dB), equal to 0.1 bel, is more commonly used.



When are logs used?

The Richter scale measures earthquake intensity on a base -10 logarithmic scale.

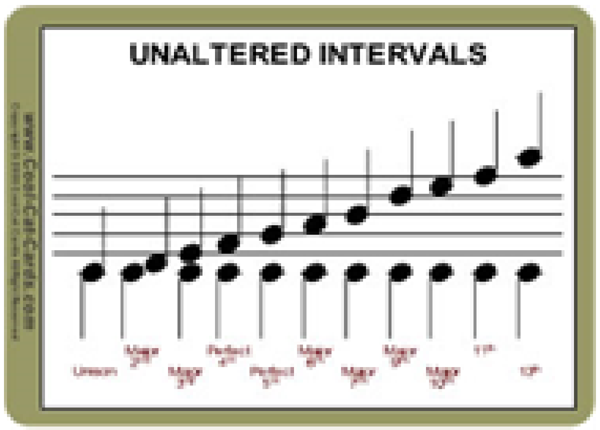


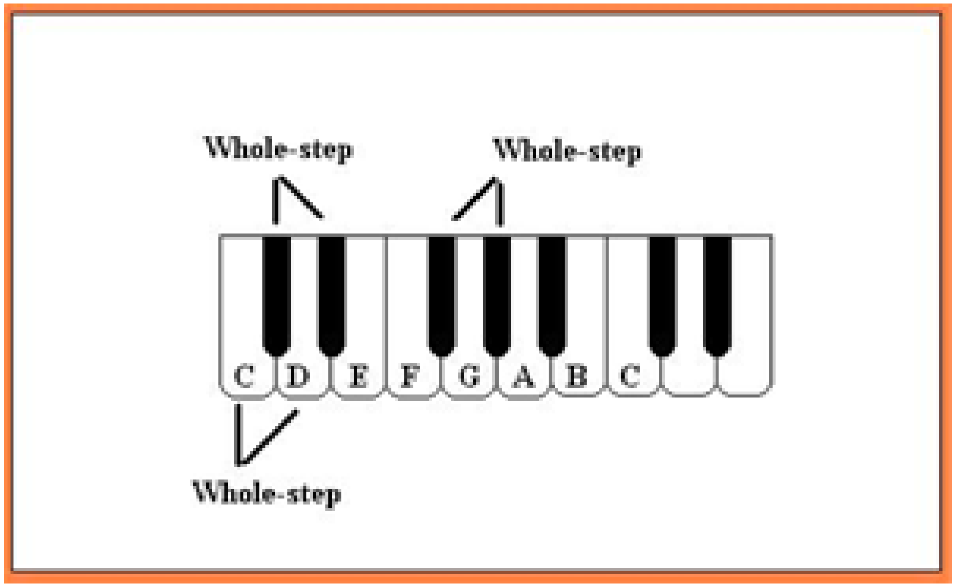
When are logs used?

In astronomy, the apparent magnitude measures the brightness of stars logarithmically, since the eye responds approximately logarithmically to brightness.

When are logs used?

Musical intervals are measured logarithmically as semitones (half steps up or down).





By the end of the lesson, we will be able to:

~ Solve log equations

~ Solve equations with more than one log

By the end of the lesson, we will be able to:

~ Solve application problems with exponentials and logs

*Exponential Growth problems

*Exponential Decay problems

Homework:

Journal 47 + Journal 48
(Skip #5-10 - you already did them.)

Assignment 48

Skip Assign 47.