Lesson 48: Exponential Growth and Decay

By the end of the lesson, we will be able to:

~ Solve application problems with exponentials and logs

*Exponential Growth problems

*Exponential Decay problems

Real life situations involving exponential growth or decay can be modeled using the equation:

Exponential Growth and Decay:

$$y = Pe^{rt}$$

where y is final amount, P is initial amount (Principal), r is the growth rate, and t is time.

decimal

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Examples of exponential growth include:

~Amount of money in an account with interest

~Population growth

~Appreciation or depreciation of property
 up   down
values

~Radioactive decay of elements.

$$y = Pe^{rt}$$

Examples:

A. Your parents put $2000 in a college fund when you are born. The account pays 5% interest. How much do you have in the account when you turn 18?

$y = ?$

$P = 2000$

5% $r = .05$

$t = 18$

$y = 2000 e^{(.05 * 18)}$

$$\boxed{y = \$4919.21}$$

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This equation $y = Pe^{rt}$ is used when we are compounding interest continually (always).

But sometimes, we only compound interest annually (once a year). We use this equation when we do that: $y = P(1 + r)^t$

$$(1+r) - 1 = r$$

In this equation $y = P(1 + r)^t$, r stands for rate.

~ If r is positive, we are growing.

~ If r is negative, we are decaying (decreasing).

$$y = P(1 + r)^t$$

Examples: Give the percent growth or decay represented by the following equation.

B. $y = 3000(1.65)^t$

$\uparrow$

$1.65 - 1 = .65$

growth of 65%

$$y = P(1 + r)^t$$

Examples: Give the percent growth or decay represented by the following equation.

c. $y = 4000(.75)^t$

$$.75 - 1 = \boxed{-.25}$$

$$\boxed{\text{decay by } 25\%}$$

$$y = P(1 + r)^t$$

Examples: Give the percent growth or decay represented by the following equation.

D. $y = 3500 \left(\dfrac{3}{4}\right)^t$

$\dfrac{3}{4} = .75$

$.75 - 1 = -.25$

decay by 25%

When trying to find growth rate or time (r or t), logarithms must be used to solve the equation.

Since the equations use e, it is easiest to use natural log (Ln) to solve.

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$$y = Pe^{rt}$$

Examples:

E. If a city population is about 112,000 people now, and if the population grows continuously at an annual rate of 4%, How long will it take to reach 250,000 people?

$y = 250,000$

$P = 112,000$

$r = .04$

$t = ?$

$$\frac{250,000}{112,000} = \frac{112000 \, e^{(.04t)}}{112,000}$$

$$\ln\left(\frac{250}{112}\right) = \ln\left(e^{.04t}\right)$$

$$\ln e \approx 1$$
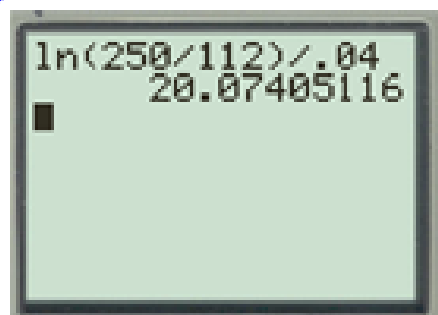
$$\frac{\ln\left(\frac{250}{112}\right)}{.04} = \frac{.04 \, t \, \ln e}{.04}$$

$$t = \frac{\ln\left(\frac{250}{112}\right)}{.04} = \boxed{20.07 \, yrs}$$

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ln(250/112)/.04
        20.07405116
■
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$$y = Pe^{rt}$$

Examples:

F. The Diaz family bought a new house 20 years ago for $80,000. The house is now worth $250,000. Assuming a steady rate of growth, what was the yearly rate of appreciation?
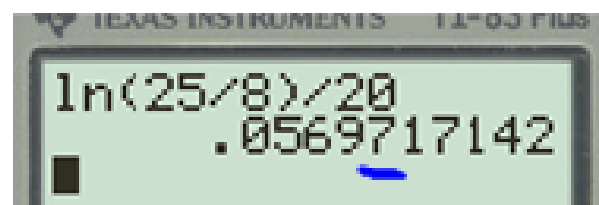
$y = 250,000$

$P = 80,000$

$r = ?$

$t = 20$

$$\frac{250,000}{80,000} = \frac{80,000\, e^{r(20)}}{80,000}$$

$$\ln\left(\frac{25}{8}\right) = \ln\left(e^{20r}\right)$$

$$\frac{\ln\left(\frac{25}{8}\right)}{20} = \frac{20r}{20} \ln e$$

$$r = \frac{\ln\left(\frac{25}{8}\right)}{20} = .0570 \quad \boxed{5.7\%}$$

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ln(25/8)/20
          .0569717142
```

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$$y = Pe^{rt}$$

Examples:

G.   How long will it take a sum of money to double if the interest rate is 4%?

$y = 2x$

$P = x$

$r = .04$

$t = ?$

$2\cancel{x} = \cancel{x} \, e^{.04t}$

$\ln(2) = \cancel{\ln}\cancel{e}^{(.04t)}$

$\dfrac{\ln(2)}{.04} = \dfrac{.04t \, \cancel{\ln e}}{.04}$

$t = \dfrac{\ln(2)}{.04} = \boxed{17.33 \text{ yrs}}$

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ln(2)/.04
            17.32867951
```

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Another application of exponential equations is **radioactive decay**. Radioactive elements decay over time, and the rate of decay is constant. Instead of a decay rate (r), the decay is often given as the **half life** of the substance. Half life is the amount of time it takes for half of the mass to decay.

When given the half life of an element, the decay rate (r) can be found using the formula:

★Memorize!

Decay rate when given half life:

$$r = \frac{-\ln 2}{half\ life}$$

$$y = Pe^{rt}$$

Examples:

Element X has a half-life of 2000 years. A sample starts with 830 grams.
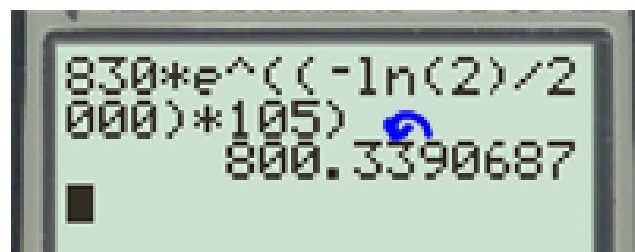
H. How many grams will be left in 105 years?

$y = ?$

$P = 830$

$r = \dfrac{-\ln(2)}{2000}$

$t = 105$

$$y = 830\, e^{\left(-\frac{\ln(2)}{2000} * 105\right)}$$

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830*e^((-ln(2)/2
000)*105)
        800.3390687
```

$y = 800.34$ grams

$$r = \dfrac{-\ln 2}{half\ life}$$

$$y = Pe^{rt}$$

Examples:

Element X has a half-life of 2000 years. A sample starts with 830 grams.

I. How long until 200 grams remain?

$$\left(-\frac{\ln(2)}{2000} t\right)$$

$y = 200$

$P = 830$

$r = \longrightarrow \dfrac{-\ln(2)}{2000}$

$t = ?$

$$\dfrac{200\!\!\!/}{830\!\!\!/} = \dfrac{830\,e}{830}$$

$$\left(-\frac{\ln(2)}{2000} t\right)$$

$$\ln \dfrac{20}{83} = \ln e$$

$$\ln\left(\dfrac{20}{83}\right) = \dfrac{-\ln(2)}{2000} t \cdot \ln e$$

$$r = \dfrac{-\ln 2}{half\ life}$$

$$y = Pe^{rt}$$

Examples: work cont...

Element X has a half-life of 2000 years. A sample starts with 830 grams.
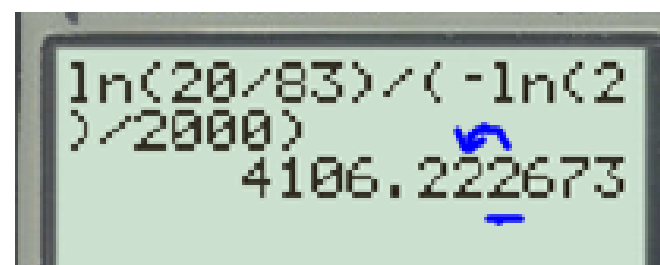
I. How long until 200 grams remain?

$y =$

$P =$

$r =$

$t =$

$$\dfrac{\ln\left(\frac{20}{83}\right)}{\frac{-\ln(2)}{2000}} = \dfrac{\frac{-\ln(2)}{2000}\,t}{\frac{-\ln(2)}{2000}}$$

$$t = \dfrac{\ln\left(\frac{20}{83}\right)}{\left(\frac{-\ln(2)}{2000}\right)} = \boxed{4106.22 \text{ yrs}}$$

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ln(20/83)/(-ln(2
)/2000)
           4106.222673
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$$r = \dfrac{-\ln 2}{half\ life}$$

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~ Solve application problems with exponentials and logs

*Exponential Growth problems

*Exponential Decay problems

Homework:

Assignment 48