Lesson 50 & 52: Degrees, Radians, & Coterminal Angles AND Arc Length & Sector Area

By the end of the lesson, we will be able to:

~ Convert Degrees to Radians

~ Convert Radians to Degrees

~ Understand and use the terms: Initial Side , Terminal Side , Standard Position , and Coterminal Side

~ Find arc length
~ Find sector area

In most real-world applications, angles are measured in degrees. However, in upper-level math classes, angles are measured in **radians**. Radians are usually written in terms of π

Relationships between Radians and Degrees:

$$2\pi \text{ radians} = 360°$$

$$1 \text{ radian} = \frac{180°}{\pi} \approx 57.3° \qquad 1° = \frac{\pi \text{ radians}}{180} \approx 0.017 \text{ radians}$$

To convert between radians and degrees, you multiply by a **conversion factor**:

Convert Radians to Degrees → $\frac{180°}{\pi}$ or $\frac{\pi}{180°}$ ← Convert Degrees to Radians

CAUTION: An angle measure without a degree symbol means radians. If you want degrees, you must use °.

Examples:

A. Convert $\frac{3\pi}{4}$ radians to degrees.

B. Convert $\frac{20\pi}{3}$ radians to degrees.

CAUTION: An angle measure without a degree symbol means radians. If you want degrees, you must use °.

Examples:

C. Convert 60° to radians.

D. Convert 330° to radians.

CAUTION: An angle measure without a degree symbol means radians. If you want degrees, you must use °.

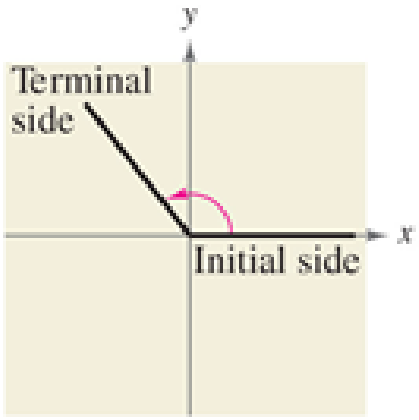
The UNIT CIRCLE is a circle with radius of 1 unit, centered at the origin of a coordinate plane.

~ Angles in STANDARD POSITION on the unit circle have their vertex at the origin.

~ One side of the angle, called the INITIAL SIDE, is on the positive x-axis.

~ The other side, called the TERMINAL SIDE, determines the measure of the angle, and is measured counterclockwise.

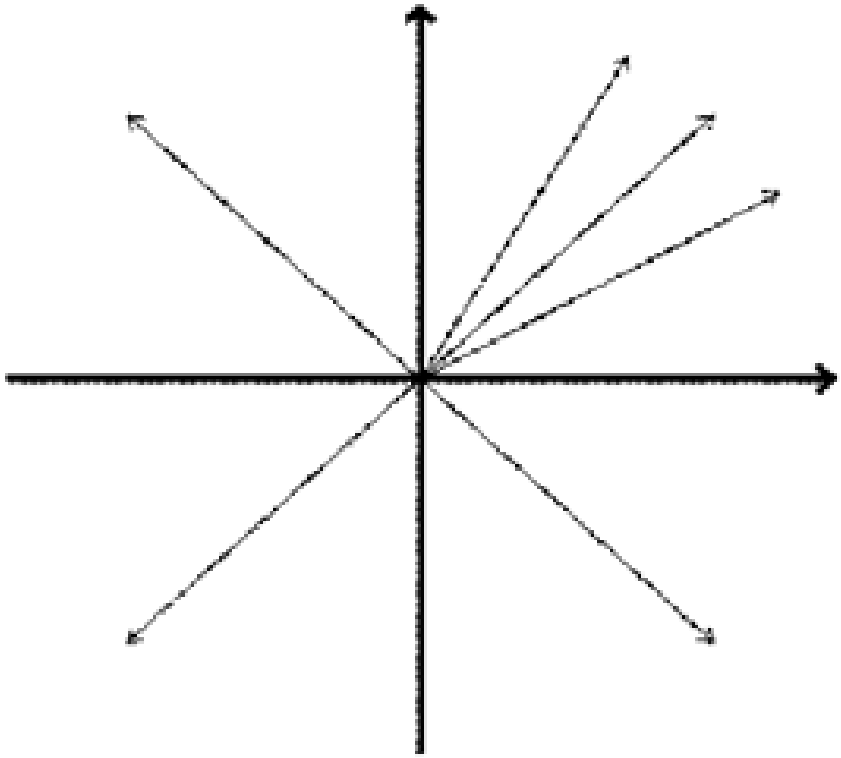
Since there is a starting direction, angles can have any measure, positive or negative. The variable for angles is usually a Greek letter, commonly θ (theta).



Angle in Standard Position

It is helpful to have some benchmark angles on the unit circle to help sketch angles.

$\frac{\pi}{6} = 30°$ $\frac{\pi}{4} = 45°$ $\frac{\pi}{3} = 60°$ $\frac{\pi}{2} = 90°$



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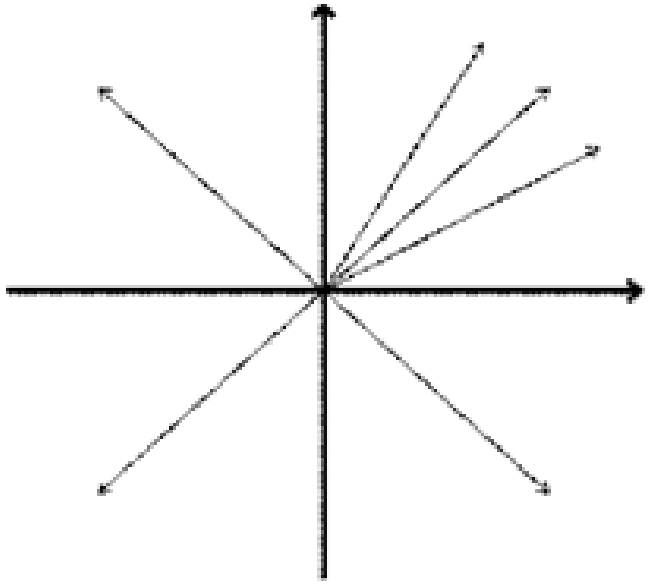
$\frac{\pi}{6} = 30°$ $\frac{\pi}{4} = 45°$ $\frac{\pi}{3} = 60°$ $\frac{\pi}{2} = 90°$

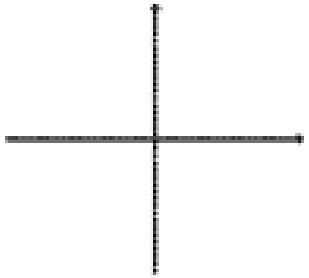
$\frac{3\pi}{4} = 135°$ $\pi = 180°$ $\frac{5\pi}{4} = 225°$

$\frac{3\pi}{2} = 270°$ $\frac{7\pi}{4} = 315°$ $2\pi = 360°$

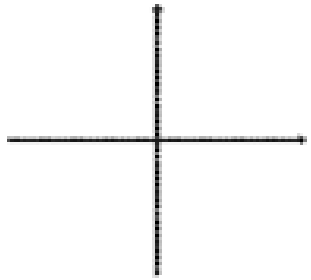


Examples: Sketch each angle in standard position.

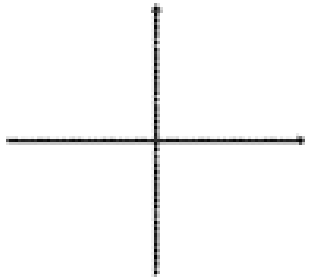
E. 50°



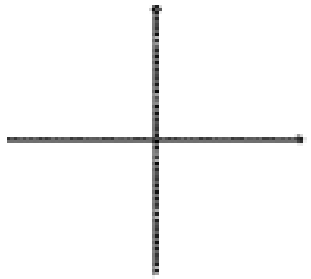
F. 210°



G. −45°



H. 630°



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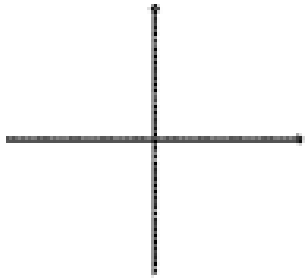
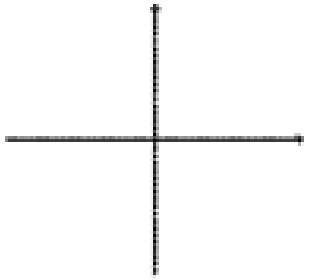
Examples: Sketch each angle in standard position.

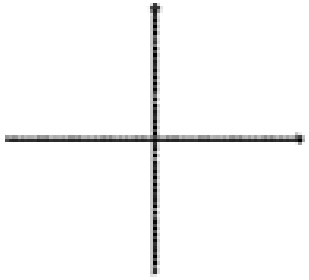
I. $\frac{\pi}{2}$



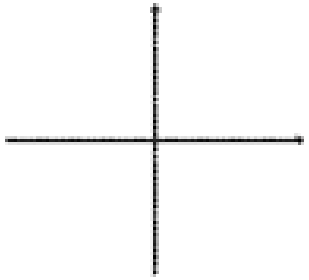
J. $\frac{3\pi}{4}$



K. $\frac{13\pi}{6}$



L. $-\frac{\pi}{6}$



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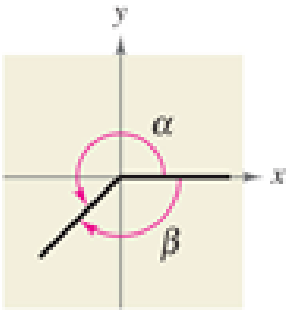
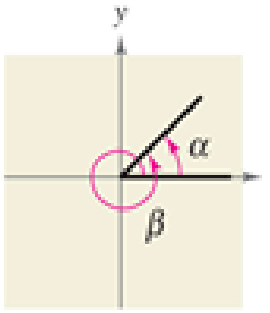
Angles with the same terminal side are called **coterminal**. Coterminal angles are always 360° or 2π greater than or less than each other (or multiples of 360° and 2π). There are infinitely many coterminal angles for a given angle. Why?





Examples: Find one positive and one negative coterminal angle.

M. 30°

N. −400°

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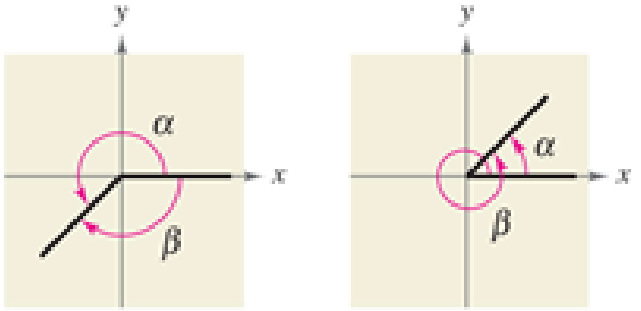


Examples: Find one positive and one negative coterminal angle.

O. $\frac{3\pi}{2}$

P. $-\pi$

Arc Length and Sectors

In geometry, you learned formulas to find arc length and sector area when the angle is in degrees. However, the formulas for arc length and sector area are much simpler when measuring in radians.

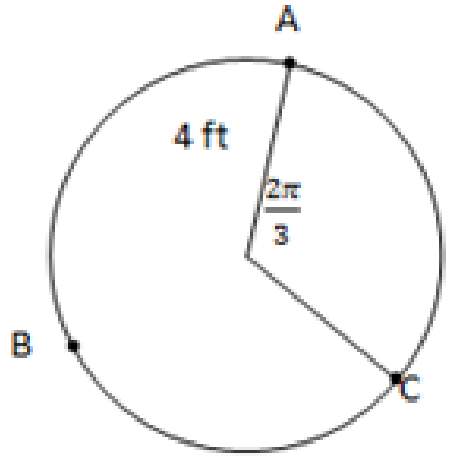
Arc length when θ is in radians: $s = r\theta$

Sector area when θ is in radians: $A = \frac{1}{2}r^2\theta$

Lesson 52: Arc Length and Sector Area

Examples:

A. Find the length of $\widehat{AC}$



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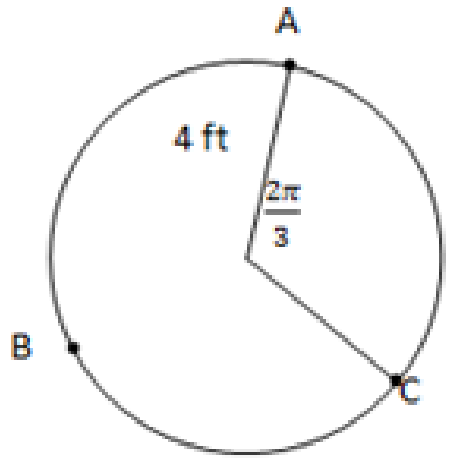
Examples:

B. Find the length of $\overset{\frown}{ABC}$



Examples:

C. Find the length of the arc of a circle with a radius of 7 cm and a central angle of $\frac{4\pi}{25}$

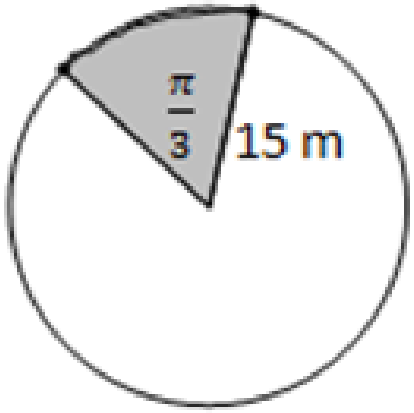
Examples: Given the radius of a circle and an arc length, find the central angle, theta. (Keep in fraction form.)

D. Radius = 4 ft and Arc Length = 20 ft

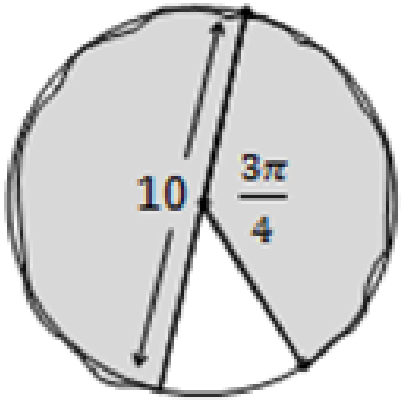
Examples: Given the radius of a circle and an arc length, find the central angle, theta. (Keep in fraction form.)

E. Radius = 5 in and Arc Length = 7 in

Example F: Find the area of the shaded sector:

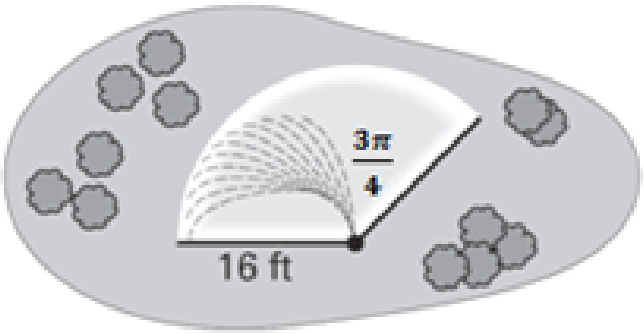


Example G: Find the area of the shaded sector:



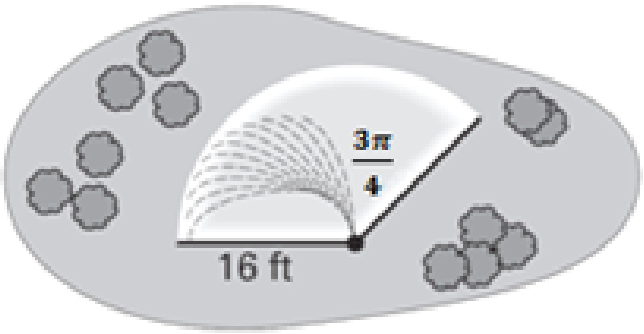
Example H: Find the area of the shaded sector:

The diagram shows the area of a lawn covered by a water sprinkler. What is the area of the lawn covered by the sprinkler?



Example 1: Find the area of the shaded sector:

The water pressure is weakened so that the radius is 10 feet. What is the area of the lawn that will be covered?



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~ Find arc length
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Can you?

Homework:

Assignment 50 (all - but it's short) &
Assignment 52 "new" part

AND

Review for Test 12