

By the end of the lesson, we will be able to:

- ~ Convert Degrees to Radians
- ~ Convert Radians to Degrees
- ~ Understand and use the terms: Initial Side, Terminal Side, Standard Position, and Coterminal Side

Lesson 50: Degrees, Radians, & Coterminal Angles

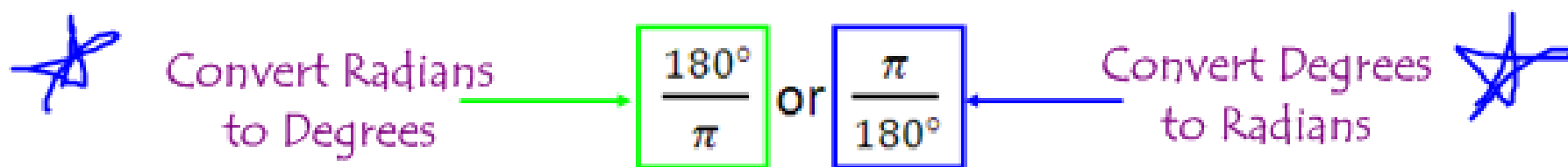
In most real-world applications, angles are measured in degrees. However, in upper-level math classes, angles are measured in radians. Radians are usually written in terms of π

Relationships between Radians and Degrees:

$$2\pi \text{ radians} = 360^\circ$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57.3^\circ \qquad 1^\circ = \frac{\pi \text{ radians}}{180} \approx 0.017 \text{ radians}$$

To convert between radians and degrees, you multiply by a **conversion factor**:



CAUTION: An angle measure without a degree symbol means radians. If you want degrees, you must use °.

Examples: mult by $\frac{180}{\pi}$

A. Convert $\frac{3\pi}{4}$ radians to degrees.

$$\frac{\cancel{3\pi}^1}{\cancel{4}_1} \cdot \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}_1} = \frac{3(45)}{1} = \boxed{135^\circ}$$

$$\frac{45}{135}$$

B. Convert $\frac{20\pi}{3}$ radians to degrees.

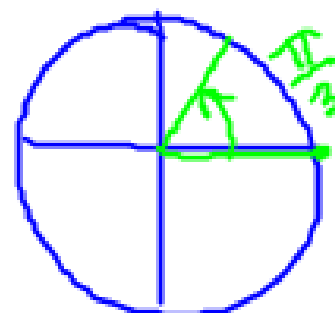
$$\frac{\cancel{20\pi}^1}{\cancel{3}_1} \cdot \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}_1} = \frac{20(60)}{1} = \boxed{1200^\circ}$$

CAUTION: An angle measure without a degree symbol means radians. If you want degrees, you must use $^\circ$.

Examples: $\ast \frac{\pi}{180^\circ}$

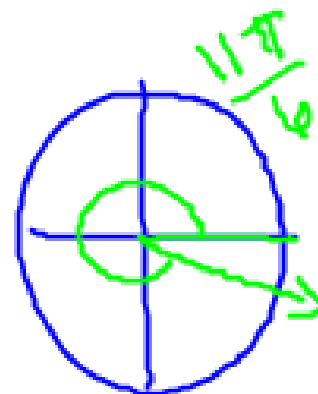
C. Convert 60° to radians.

$$\frac{\cancel{60}}{1} \cdot \frac{\pi}{\cancel{180}_3} = \boxed{\frac{\pi}{3}}$$



D. Convert 330° to radians.

$$\frac{\cancel{330}}{1} \cdot \frac{\pi}{\cancel{180}_6} = \boxed{\frac{11\pi}{6}}$$



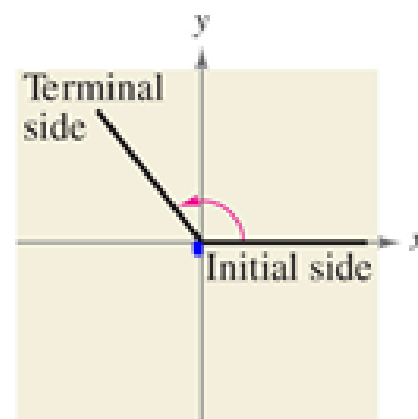
CAUTION: An angle measure without a degree symbol means radians. If you want degrees, you must use $^\circ$.

Lesson 50: Degrees, Radians, & Coterminal Angles

The **UNIT CIRCLE** is a circle with radius of 1 unit, centered at the origin of a coordinate plane.

- ~ Angles in **STANDARD POSITION** on the unit circle have their vertex at the origin.
- ~ One side of the angle, called the **INITIAL SIDE**, is on the positive x-axis.
- ~ The other side, called the **TERMINAL SIDE**, determines the measure of the angle, and is measured counterclockwise.

Since there is a starting direction, angles can have any measure, positive or negative. The variable for angles is usually a Greek letter, commonly θ (theta).



Angle in Standard Position

Lesson 50: Degrees, Radians, & Coterminal Angles

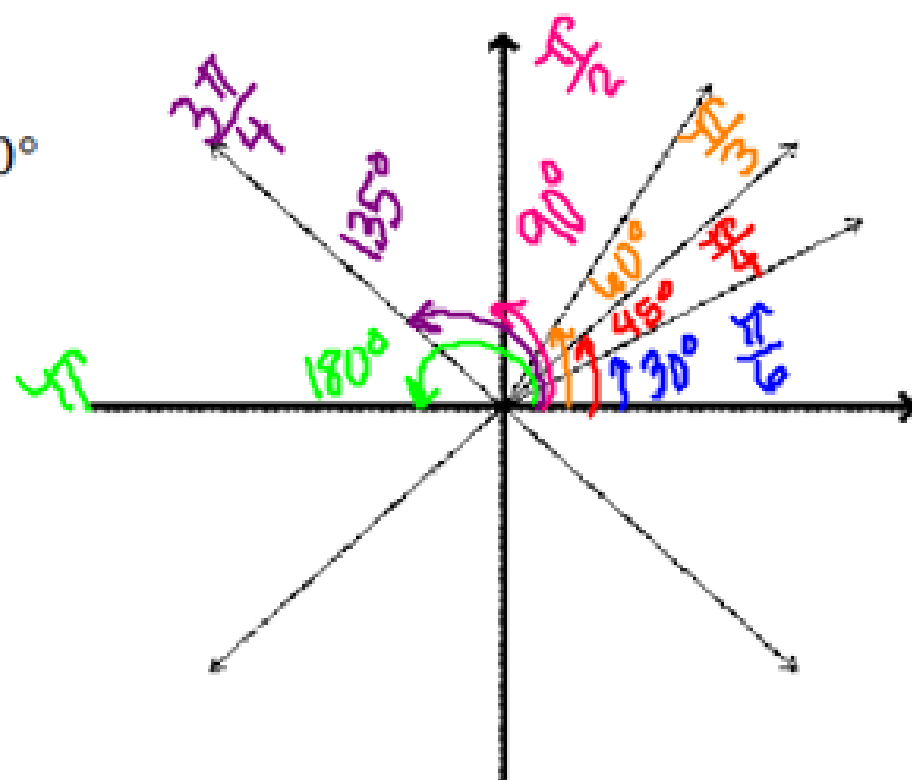
It is helpful to have some benchmark angles on the unit circle to help sketch angles.

$$\frac{\pi}{6} = 30^\circ$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{3} = 60^\circ$$

$$\frac{\pi}{2} = 90^\circ$$



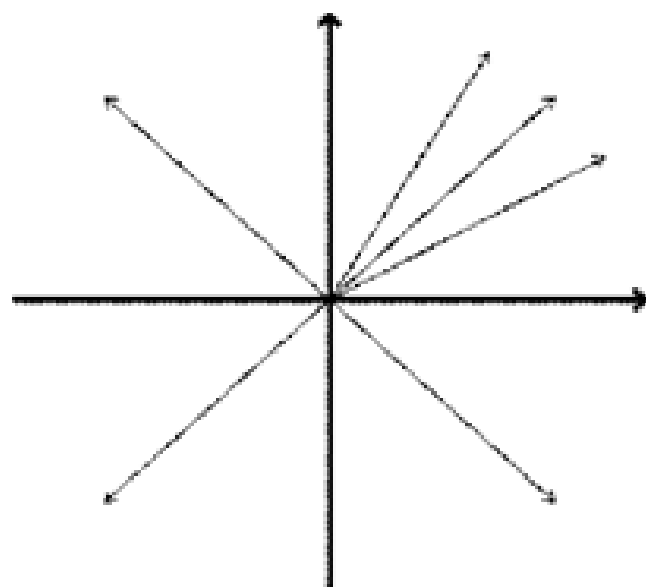
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It is helpful to have some benchmark angles on the unit circle to help sketch angles.

$$\frac{\pi}{6} = 30^\circ \quad \frac{\pi}{4} = 45^\circ \quad \frac{\pi}{3} = 60^\circ \quad \frac{\pi}{2} = 90^\circ$$

$$\frac{3\pi}{4} = 135^\circ \quad \pi = 180^\circ \quad \frac{5\pi}{4} = 225^\circ$$

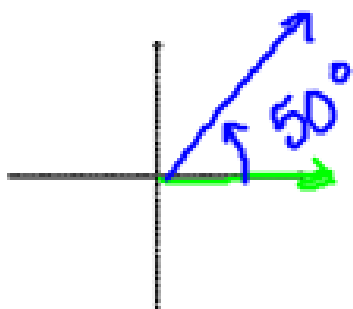
$$\frac{3\pi}{2} = 270^\circ \quad \frac{7\pi}{4} = 315^\circ \quad 2\pi = 360^\circ$$



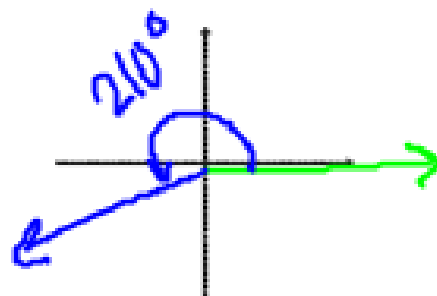
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Examples: Sketch each angle in standard position.

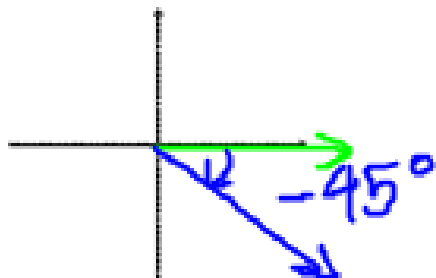
E. 50°



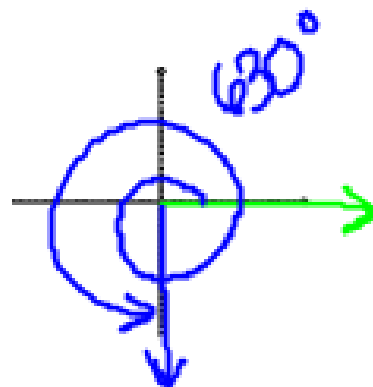
F. 210°



G. -45°



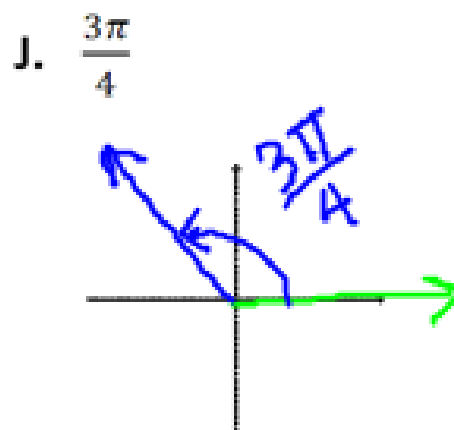
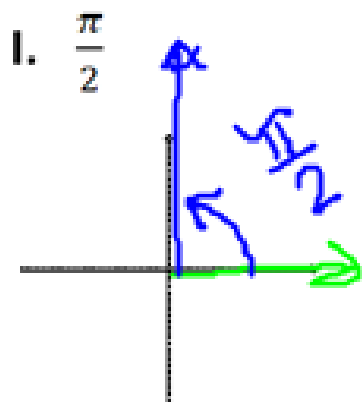
H. 630°



$$\begin{array}{r} 5 \times 30 \\ -360 \\ \hline 270 \end{array}$$

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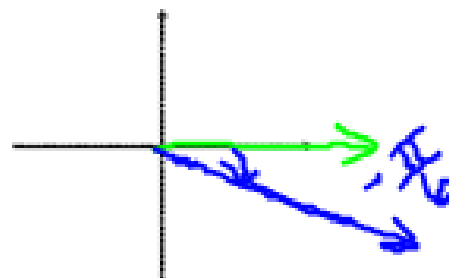
Examples: Sketch each angle in standard position.



K. $\frac{13\pi}{6} = \frac{12\pi}{6} + \frac{\pi}{6} = 2\pi + \frac{\pi}{6}$

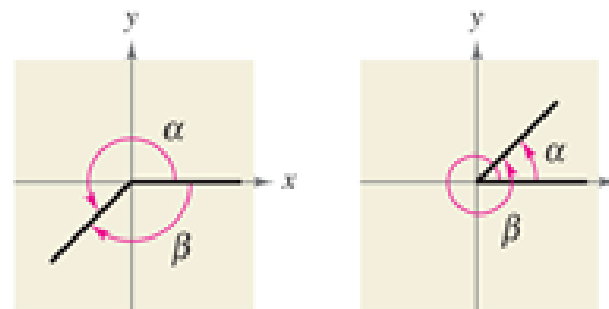


L. $-\frac{\pi}{6}$



Lesson 50: Degrees, Radians, & Coterminal Angles

Angles with the same terminal side are called coterminal. Coterminal angles are always 360° or 2π greater than or less than each other (or multiples of 360° and 2π). There are infinitely many coterminal angles for a given angle. Why?



Examples: Find one positive and one negative coterminal angle.

M. 30°

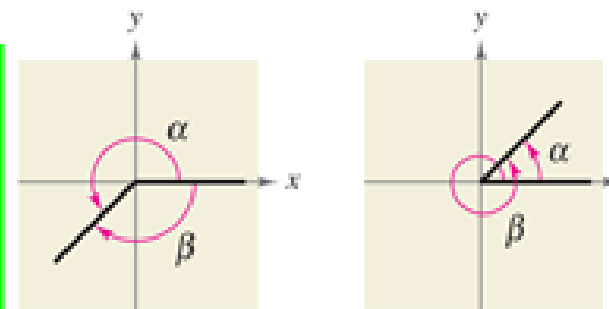
$$30^\circ + 360^\circ = \boxed{390^\circ}$$
$$30^\circ - 360^\circ = \boxed{-330^\circ}$$

N. -400°

$$-400 + 360 = \boxed{-40^\circ}$$
$$-400 - 360 = -760^\circ$$
$$-400 + 360 + 360 =$$
$$-40 + 360 = \boxed{320^\circ}$$

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Angles with the same terminal side are called **coterminal**. Coterminal angles are always 360° or 2π greater than or less than each other (or multiples of 360° and 2π). There are infinitely many coterminal angles for a given angle. Why?



Examples: Find one positive and one negative coterminal angle.

O. $\frac{3\pi}{2}$

$$\frac{3\pi}{2} + \frac{2\pi}{1 \cdot 2} = \frac{3\pi}{2} + \frac{4\pi}{2} = \boxed{\frac{7\pi}{2}}$$

$$\frac{3\pi}{2} - \frac{4\pi}{2} = \boxed{-\frac{\pi}{2}}$$

P. $-\pi$

$$-\pi - 2\pi = \boxed{-3\pi}$$

$$-\pi + 2\pi = \boxed{\pi}$$

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can you?

Homework:

Assignment 50
&
Test Review 12