Lesson 51: Unit Circle

By the end of the lesson, we will be able to:

~ Use the Unit Circle to find exact values of

- sine
- cosine
- tangent

Lesson 51: Unit Circle

The unit circle has another use – it's a quick reference to find the sine or cosine values for the common angles. The common angles we use include $30°,\ 45°, 60°, 90°, 120°, 135°, 150°, 180°, etc.$.

The rays that form the outside of the angles intersect the circle in specific coordinate points, and these points represent the sine and cosine values of the angle itself.

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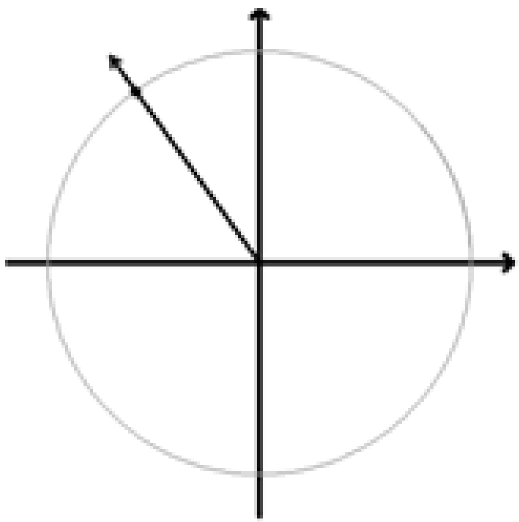
When an angle is in standard position, you can use points on the unit circle to find trigonometric ratios for the angle.



Points (x, y) on the unit circle:

$\sin\theta = y$ $\cos\theta = x$ $\tan\theta = \frac{y}{x}$

$(\cos(\theta), \sin(\theta))$

x y

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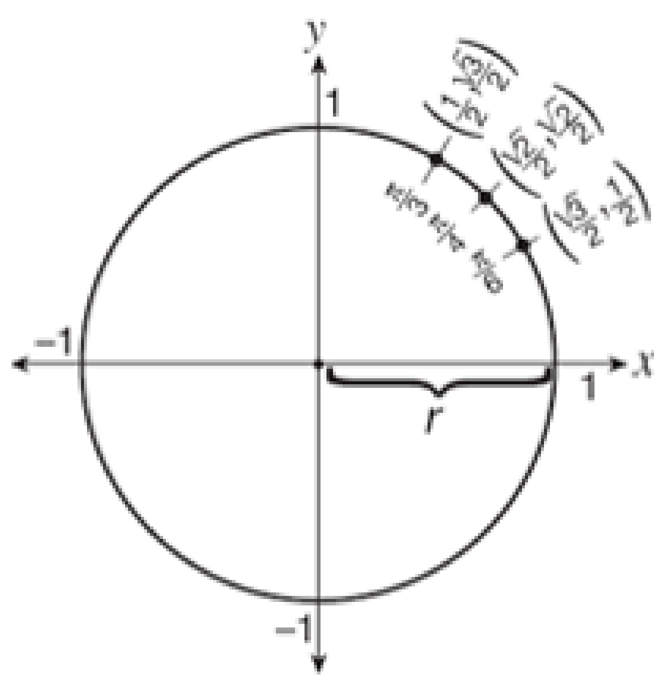
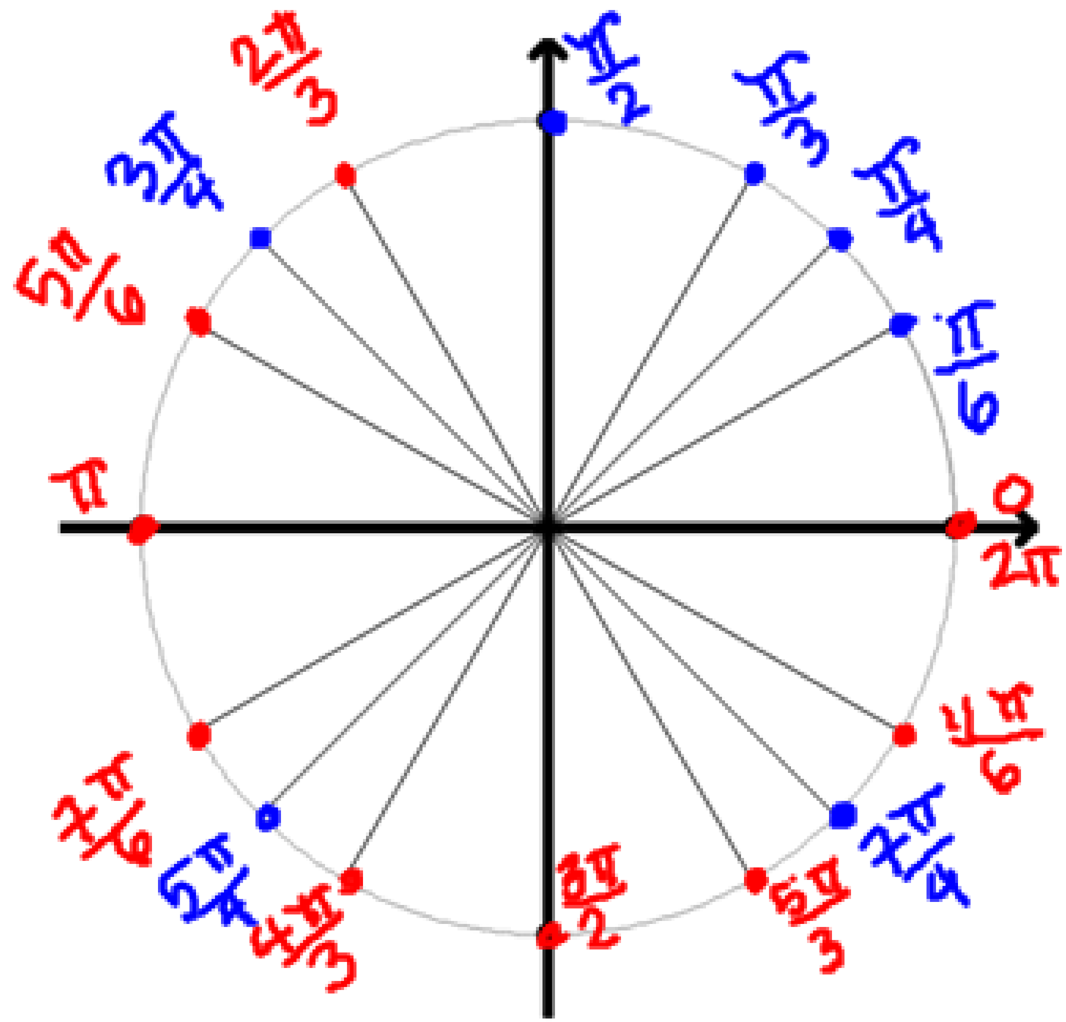
Using trigonometry, we can find points on the unit circle at benchmark angles. We can then use these coordinates to find the exact values of trigonometric functions for any given angle.

Unit Circle





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Special Right Triangles

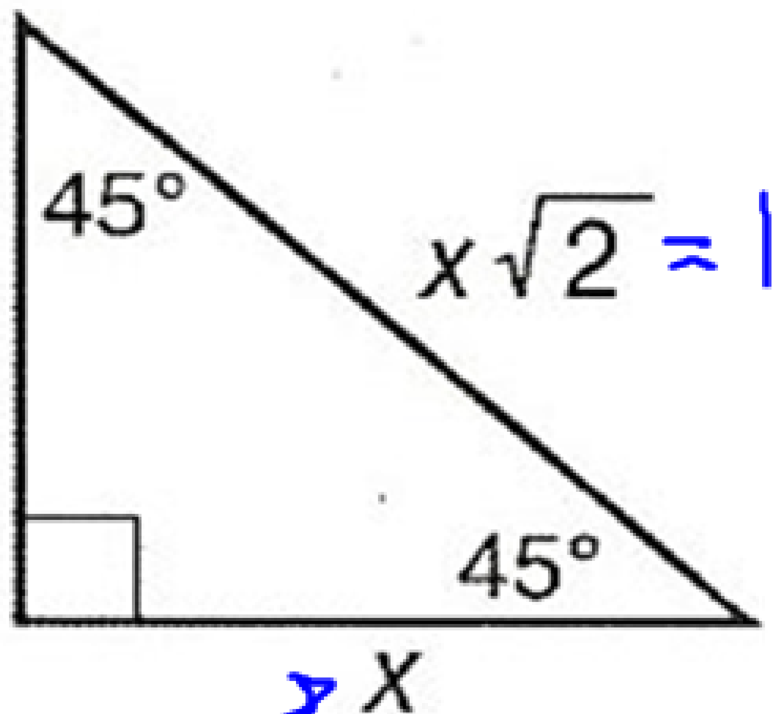
45°- 45°- 90° Triangle Theorem

Both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg. radius = 1

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{\sqrt{2}}{2}$$



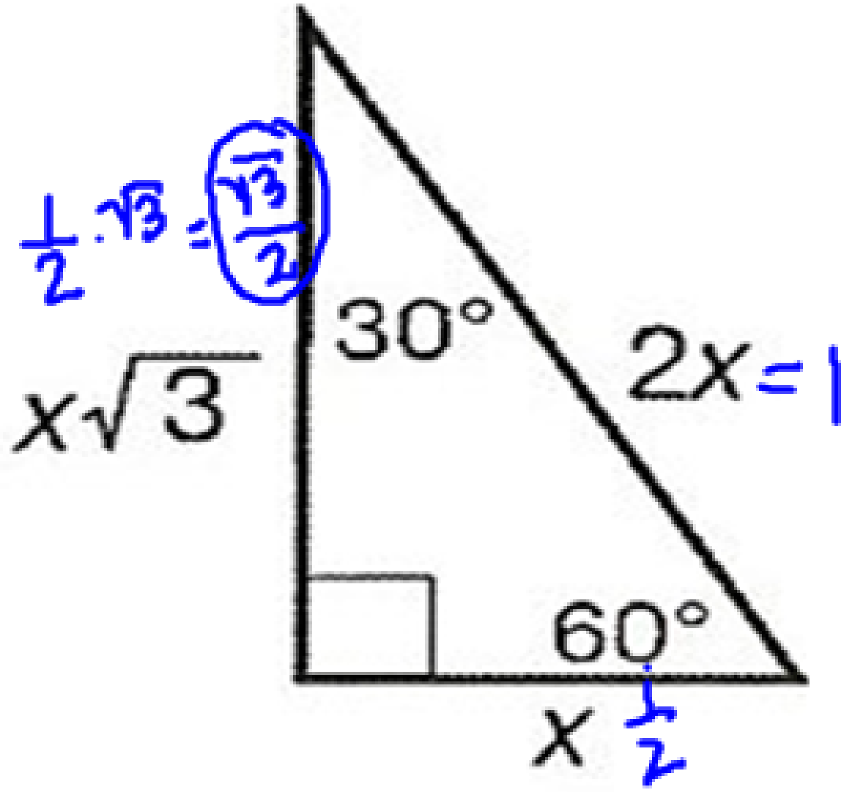
Special Right Triangles

30°- 60°- 90° Triangle Theorem

The length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the shorter leg.

$\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$



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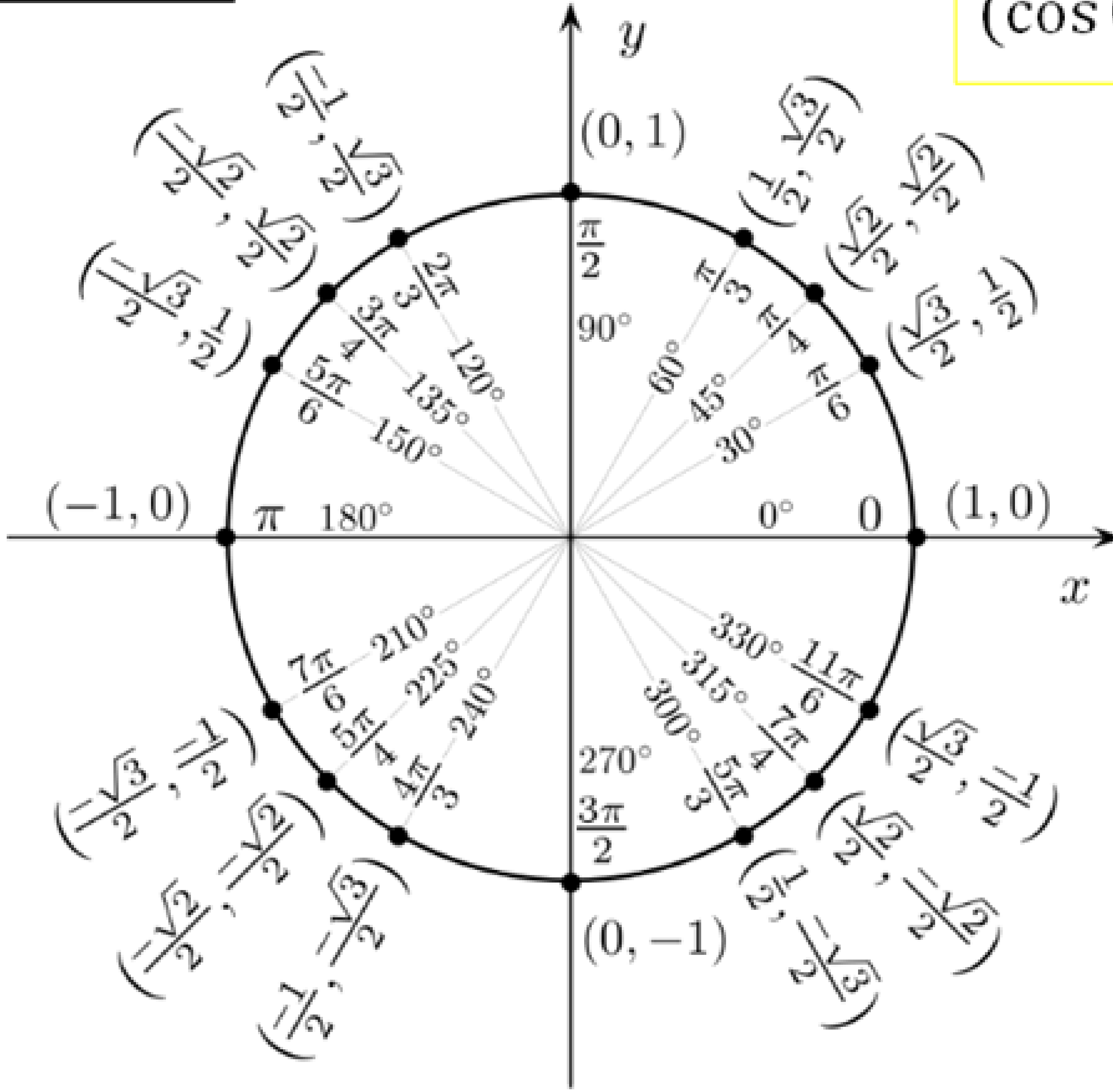
$(\cos\theta, \sin\theta)$



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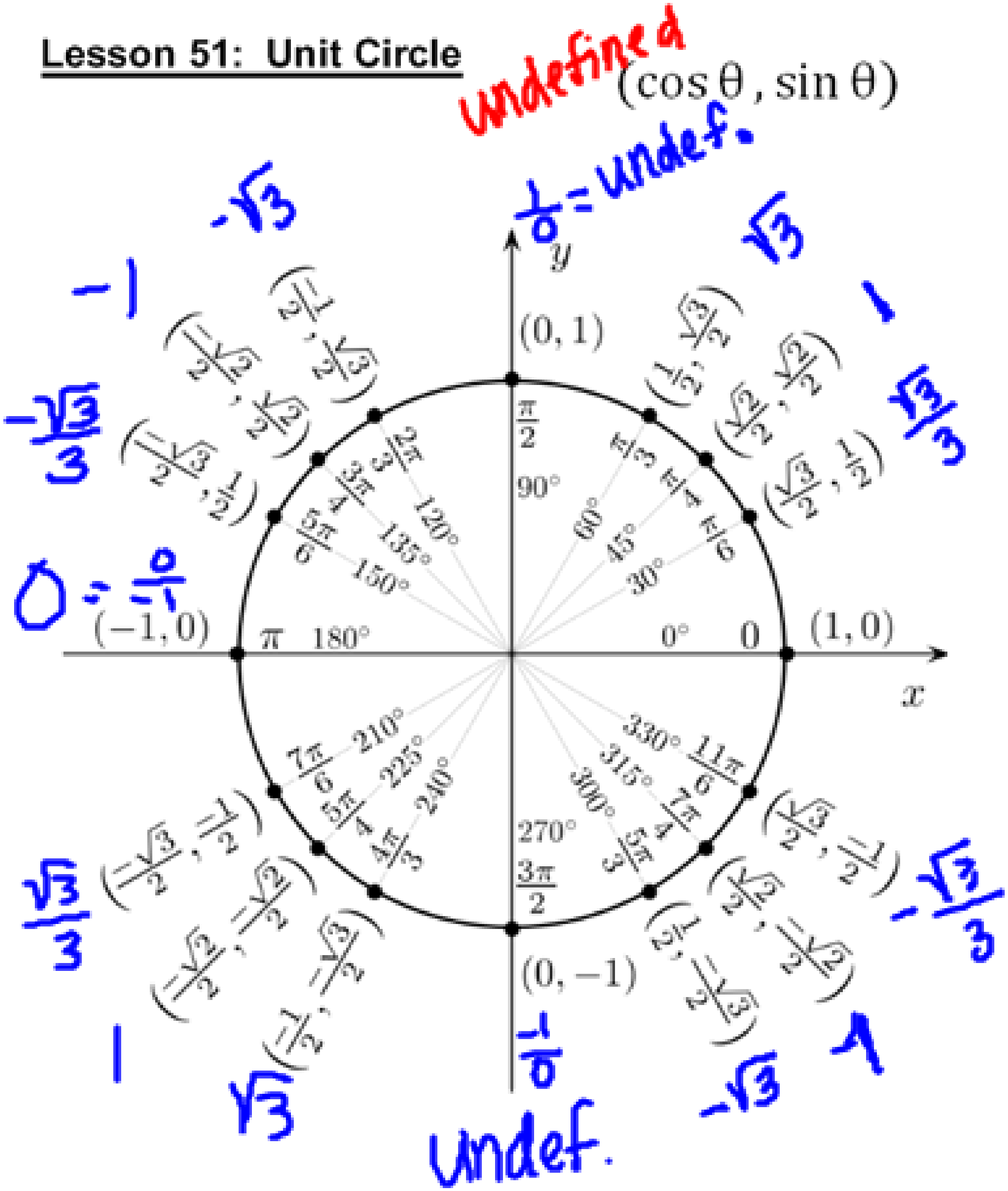

$$\tan(\theta) = \frac{y}{x}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan\left(\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{2}{\sqrt{2}}\right) = 1$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{1}\right) = \sqrt{3}$$

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$(\cos(\theta), \sin(\theta))$ — x, y

Examples: Find the exact value of each trigonometric function using the unit circle diagram.

A. $\cos 45° = \frac{\sqrt{2}}{2}$ (x)

B. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ (y)

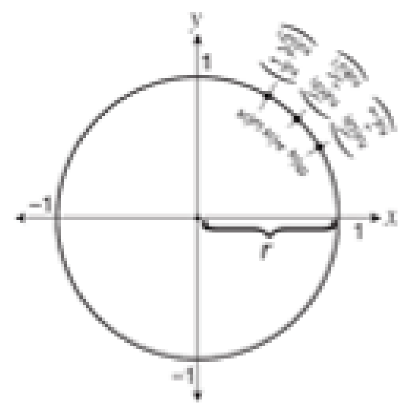
C. $\tan 240° = \sqrt{3}$ (y/x)

D. $\sin \frac{\pi}{2} = 1$ (y)

E. $\cos \frac{\pi}{2} = 0$ (x)

F. $\tan \frac{\pi}{2} =$ undef. (y/x)

Unit Circle



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Examples: Find the exact value of each trigonometric function using the unit circle diagram.

G. $\tan 315^\circ = -1$

$\frac{7\pi}{4}$

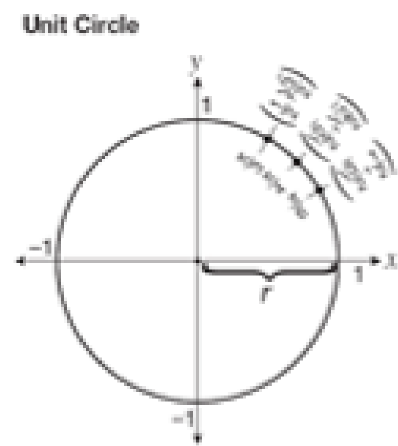
H. $\sin(-30^\circ) = -\frac{1}{2}$

$-\frac{\pi}{6} \rightarrow \frac{11\pi}{6}$

I. $\cos \frac{11\pi}{4} = -\frac{\sqrt{2}}{2}$

$\frac{8\pi}{4} + \frac{3\pi}{4}$

$\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$



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Examples: Find each angle, theta. There may be more than one answer.

J. $\sin\theta = -\frac{\sqrt{2}}{2}$ (y)

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

$\theta = 225°, 315°$

K. $\sin\theta = -1$ (y)

$\theta = \frac{3\pi}{2}$ $\theta = 270°$

L. $\cos\theta = \frac{\sqrt{3}}{2}$ (x)

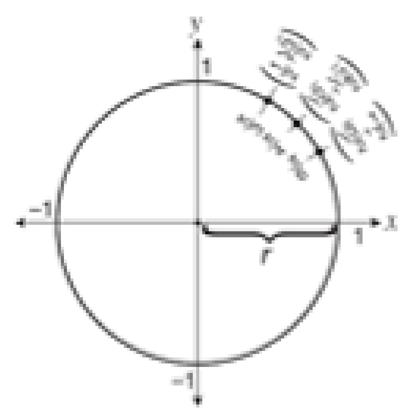
$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$ or $\theta = 30°, 330°$

M. $\tan\theta = \frac{\sqrt{3}}{3}$ ($\frac{y}{x}$)

$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

$\theta = 30°, 210°$

Unit Circle



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Mode → Radians

Examples: Find each angle, theta, in Radians on your Calculator. Round to 4 decimal places.

N. $\sin\theta = \frac{7}{8}$ → $\sin^{-1}\left(\frac{7}{8}\right) = 1.0654$

O. $\cos\theta = \frac{2}{5}$ → $\cos^{-1}\left(\frac{2}{5}\right) = 1.1593$

P. $\tan\theta = 14$ → $\tan^{-1}(14) = 1.4995$

Lesson 51: Unit Circle

By the end of the lesson, we will be able to:

~ Use the Unit Circle to find exact values of

- -sine
- -cosine
- -tangent

Can you?

Lesson 51: Unit Circle

Homework:

Assignment 51

& Unit Circle Monster