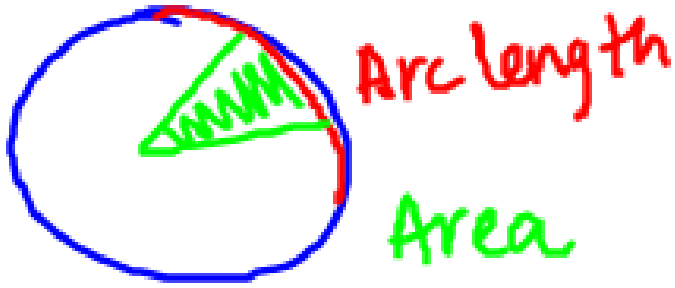
Lesson 52: Arc Length and Sector Area

By the end of the lesson, we will be able to:

~ Find arc length
~ Find sector area



Arc Length and Sectors

In geometry, you learned formulas to find arc length and sector area when the angle is in degrees. However, the formulas for arc length and sector area are much simpler when measuring in radians.

(s) (radius)

Arc length when θ is in radians: $s = r\theta$

Sector area when θ is in radians: $A = \frac{1}{2}r^2\theta$

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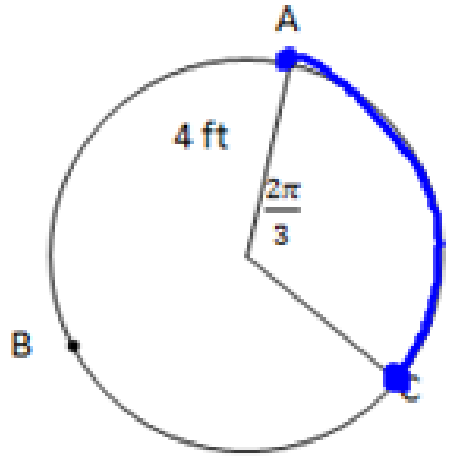
Examples: $S = r\theta$

A. Find the length of $\overset{\frown}{AC}$



$r = 4$

$\theta = \frac{2\pi}{3}$

$S = \frac{4}{1}\left(\frac{2\pi}{3}\right)$

$= \boxed{\frac{8\pi}{3} \text{ ft}}$

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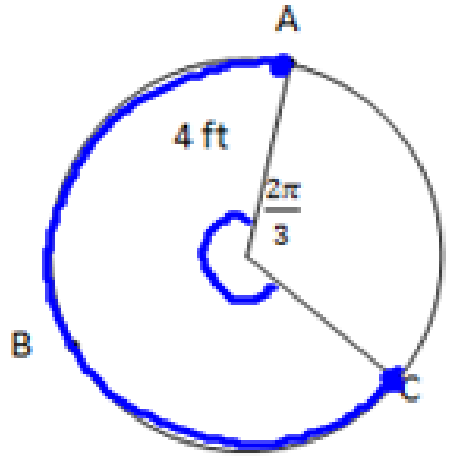
Examples:

B. Find the length of $\overset{\frown}{ABC}$



$r = 4$

$\theta = \frac{2\pi}{1} - \frac{2\pi}{3}$

$= \frac{6\pi}{3} - \frac{2\pi}{3}$

$\theta = \frac{4\pi}{3}$

$S = \frac{4}{1}\left(\frac{4\pi}{3}\right)$

$= \boxed{\frac{16\pi}{3}\text{ ft}}$

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Examples:

C. Find the length of the arc of a circle with a radius of 7 cm and a central angle of $\frac{4\pi}{25}$

r

θ

$$S = \frac{7}{1}\left(\frac{4\pi}{25}\right)$$

$$= \boxed{\frac{28\pi}{25} \text{ cm}}$$

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Examples: Given the radius of a circle and an arc length, find the central angle, theta. (Keep in fraction form.)

D. Radius = 4 ft and Arc Length = 20 ft

$$\frac{20}{4} = \frac{4\theta}{4}$$

$S = \theta$ or $\theta = S$ radians

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Examples: Given the radius of a circle and an arc length, find the central angle, theta. (Keep in fraction form.)

E. Radius = 5 in (r) and Arc Length = 7 in (s)

$$\frac{7}{s} = \frac{5\theta}{s}$$

$$\boxed{\theta = \frac{7}{5}} \text{ or}$$

$\theta = \frac{7}{5}$ radians

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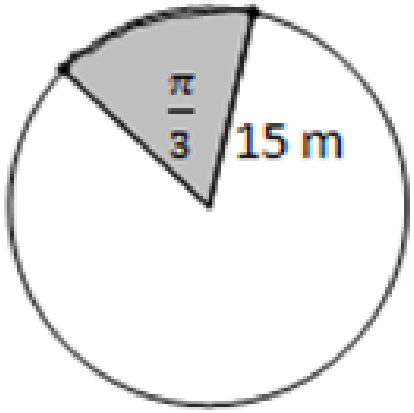
Example F: Find the area of the shaded sector:

$$A = \frac{1}{2} r^2 \theta$$



$$A = \frac{1}{2} \cdot \frac{15^2}{1} \cdot \frac{\pi}{3}$$

$$A = \frac{1 \cdot 225 \cdot \pi}{2 \cdot 1 \cdot 3}$$ (225 → 75, 3 → 1)

$$A = \boxed{\frac{75\pi}{2} \text{ m}^2}$$

$r = 15$

$\theta = \frac{\pi}{3}$

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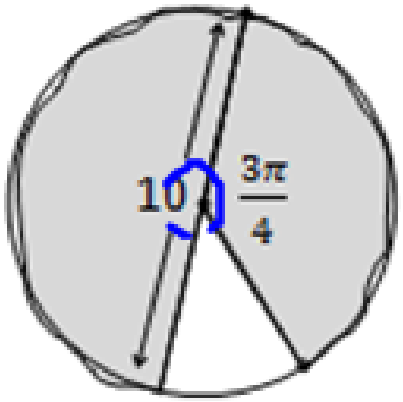
Example G: Find the area of the shaded sector:



$$A = \frac{1}{2} \cdot \frac{5^2}{1} \cdot \frac{7\pi}{4}$$

$$A = \frac{1 \cdot 25 \cdot 7\pi}{2 \cdot 1 \cdot 4}$$

$$A = \boxed{\frac{175\pi}{8} \text{ units}^2}$$

$$\theta = \pi + \frac{3\pi}{4}$$

$$\theta = \frac{4\pi}{4} + \frac{3\pi}{4}$$

$$\theta = \frac{7\pi}{4}$$

$r = 5$

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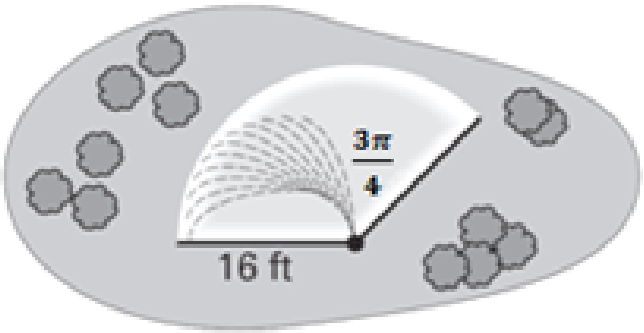
Example H: Find the area of the shaded sector:

The diagram shows the area of a lawn covered by a water sprinkler. What is the area of the lawn covered by the sprinkler?



$$A = \frac{1}{2} \cdot \frac{16^2}{1} \cdot \frac{3\pi}{4}$$

$$A = \frac{1 \cdot 256 \cdot 3\pi}{2 \cdot 1 \cdot 4}$$

$$A = 32(3\pi) = \boxed{96\pi \text{ ft}^2}$$

$$\theta = \frac{3\pi}{4}$$

$$r = 16 \text{ ft}$$

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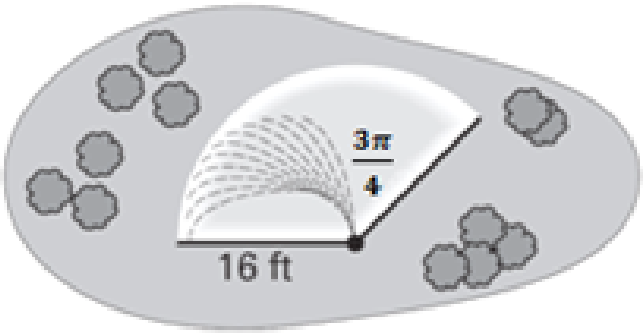
Example 1: Find the area of the shaded sector:

The water pressure is weakened so that the radius is 10 feet. What is the area of the lawn that will be covered?

$$A = \frac{1}{2} \cdot \frac{10^2}{1} \cdot \frac{3\pi}{4}$$

$$A = \frac{1 \cdot \cancel{100}^{25} \cdot 3\pi}{2 \cdot 1 \cdot \cancel{4}_{1}}$$

$$= \boxed{\frac{75\pi}{2} \text{ ft}^2}$$



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By the end of the lesson, we will be able to:

~ Find arc length
~ Find sector area

Can you?

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Homework:

Assignment 52