# Lesson 53: Graphing Trig Functions

## By the end of the lesson, we will be able to:

- ~Find the Amplitude and Period
- ~Graph Sine and Cosine functions

Lesson 53: Graphing Trig Functions

# Graphing Trig Functions

$$y = a \sin b(x - h) + k$$
$$y = a \cos b(x - h) + k$$

**Transformations of trig functions:**

- $a$ gives amplitude (vertical stretch); if $a$ is negative, the graph is flipped upside down
- $b$ gives period (horizontal stretch), use the formula: $\text{period} = \frac{2\pi}{b}$
- $h$ gives the horizontal shift, called phase shift
- $k$ gives the vertical shift

* This year, we are not going to worry about h and k.

# Lesson 53: Graphing Trig Functions

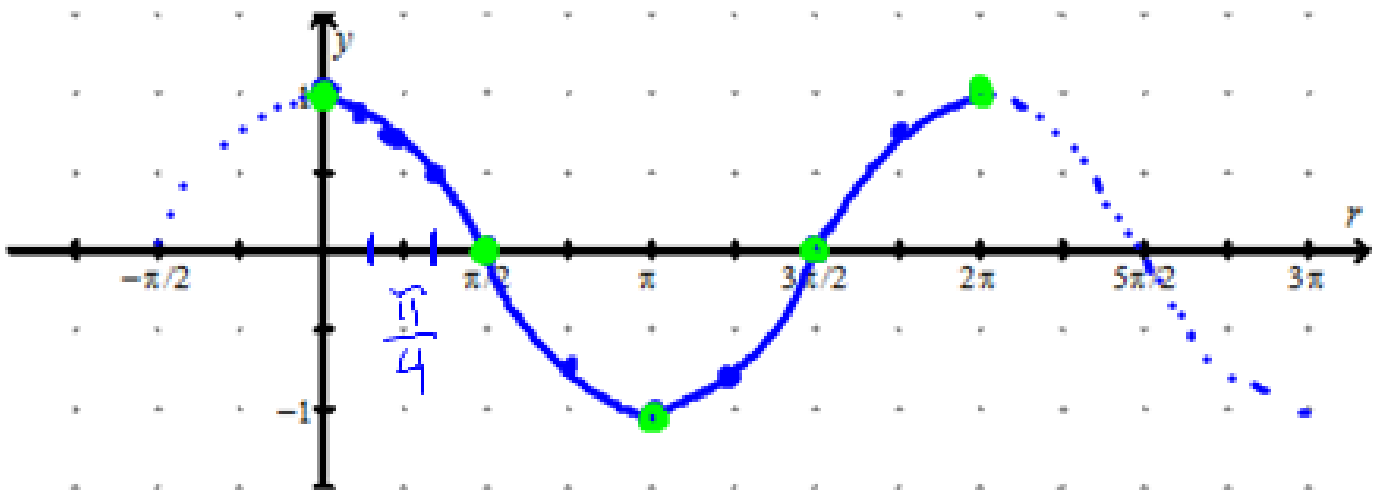
$\frac{\sqrt{3}}{2} = .86$ $\frac{\sqrt{2}}{2} = .71$

## Example 1: Graph the following trig function.

| $\theta$ (radians) | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\cos\theta$ (decimal) | 1 | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $\frac{1}{2}$ | 0 | $-\sqrt{2}/2$ | $-1$ | $-\sqrt{2}/2$ | 0 | $\sqrt{2}/2$ | 1 |

$y = \cos\theta$

$y = 1 \cdot \cos(1\theta)$




Amplitude:

$A = 1$

Period: $P = 2\pi$

$\frac{2\pi}{6} \to \frac{2\pi}{1}$

Critical Values: (in green) $(0,1)$ $(\frac{\pi}{2},0)$ $(\pi,-1)$ $(\frac{3\pi}{2},0)$ $(2\pi,1)$

Domain: $\mathbb{R}$

Range: $-1 \le y \le 1$

# Lesson 53: Graphing Trig Functions

Reminder: Critical Values for Trig Functions are the

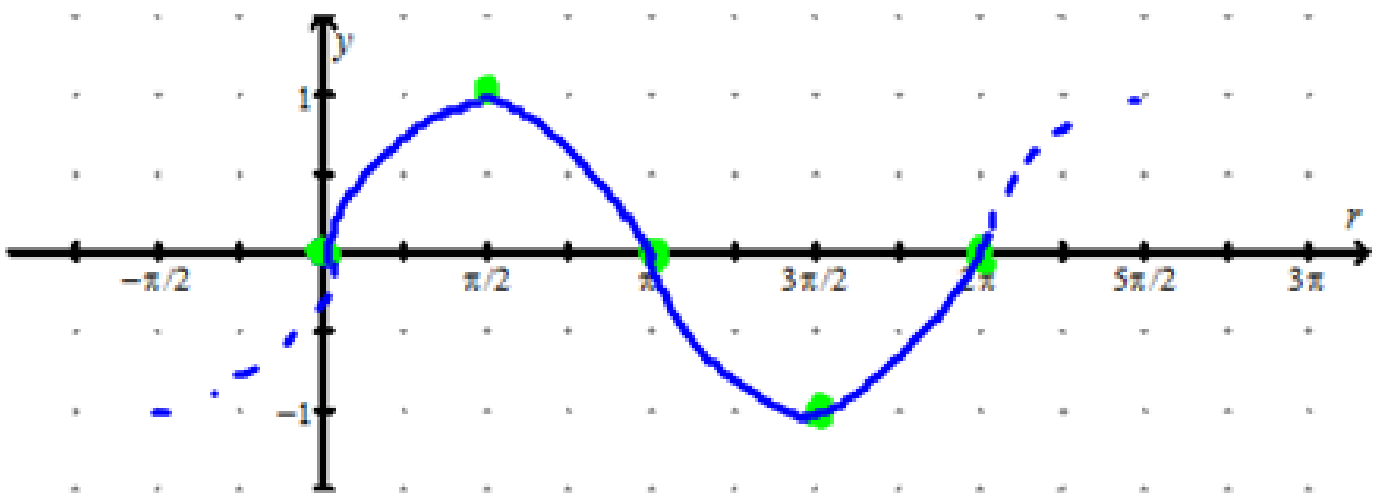
* High points on the wave.
* Low points on the wave.
* The points where the wave crosses the x-axis.

# Lesson 53: Graphing Trig Functions

## Example 2: Graph the following trig function.

| $\theta$ (radians) | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sin\theta$ (decimal) | 0 | 1/2 | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 | $\sqrt{2}/2$ | 0 | $-\sqrt{2}/2$ | -1 | $-\sqrt{3}/2$ | 0 |

$$y = \sin\theta$$




Amplitude: $A = 1$

Period: $2\pi$

$\frac{2\pi}{1}$

Critical Values: $(0,0)\left(\frac{\pi}{2},1\right)(\pi,0)\left(\frac{3\pi}{2},-1\right)(2\pi,0)$

Domain: $\mathbb{R}$

Range: $-1 \le y \le 1$

# Lesson 53: Graphing Trig Functions

A P

**Example 3:** State the Amplitude and Period of each function.

$A = |a|$

a.) $y = 3\cos\left(\frac{1}{2}\theta\right)$

$A = 3$

$A = 3$
$P = 4\pi$

$P = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = \frac{2\pi}{1} \cdot \frac{2}{1} = 4\pi$

Now graph $y = \cos\theta$ with $y = 3\cos\left(\frac{1}{2}\theta\right)$ (Use Zoom #7)

What changed? 3 times as tall

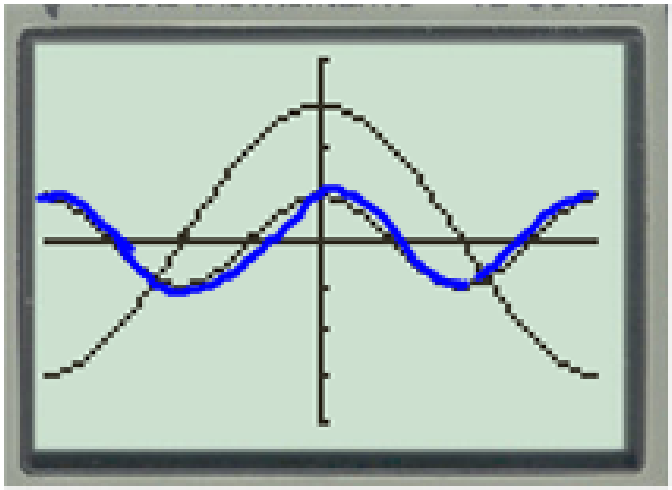
Stretched out Period (doubled)

# Lesson 53: Graphing Trig Functions

**Example 3:** State the Amplitude and Period of each function.

a.) $y = 3\cos\left(\frac{1}{2}\theta\right)$

$y = \cos\theta$



3 times as tall

Period is twice as long

Now graph $y = \cos\theta$ with $y = 3\cos\left(\frac{1}{2}\theta\right)$ (Use Zoom #7)

What changed?

# Lesson 53: Graphing Trig Functions

**Example 3:** State the Amplitude and Period of each function.

b.) $y = 2\sin\theta$

$A = 2$

$P = \frac{2\pi}{1}$

$A = 2$

$P = 2\pi$

Now graph $y = \sin\theta$ with $y = 2\sin\theta$ (Use Zoom #7)

What changed? twice as tall

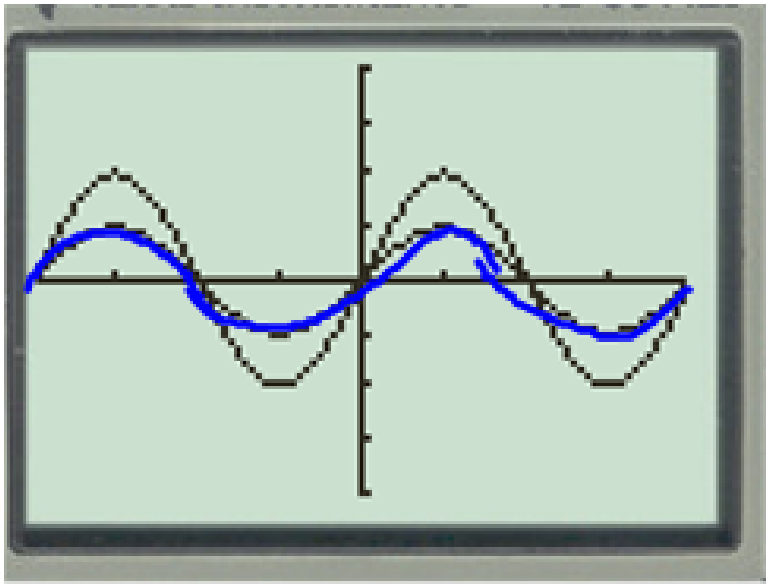
Same period

# Lesson 53: Graphing Trig Functions

**Example 3:** State the Amplitude and Period of each function.

b.) $y = 2\sin\theta$

$y = \sin\theta$



Now graph $y = \sin\theta$ with $y = 2\sin\theta$ (Use Zoom #7)

What changed?

# Lesson 53: Graphing Trig Functions

**Example 3:** State the Amplitude and Period of each function.

c.) $y = 6\cos\left(\frac{2}{3}\theta\right)$

$A = 6$

$P = \frac{2\pi}{2/3} = \frac{2\pi}{1} \cdot \frac{3}{2} = 3\pi$

$A = 6$
$P = 3\pi$

What transfomations are going to occur?

- 6 times as tall
- longer Period (1.5 times as long)

# Lesson 53: Graphing Trig Functions

**Steps to graph trig functions:**

1. Identify amplitude and calculate the period.
2. If there is a vertical shift, sketch a line at y=k, along which the graph will oscillate.
3. Identify the start and end points of one period, and then the middle and quarter points. Graph the 5 critical values.

*(Remember, if there is a flip upside-down, then the y's of the critical points change signs.)*

4. Connect the critical points with a sine/cosine wave.
5. Continue the pattern for additional periods if there is space on the graph.

# Lesson 53: Graphing Trig Functions

**Steps to get critical values:**

1. Calculate the period. This will be your end point (the x-value).
2. Divide the period by 2. This will be your middle point (the x-value).
3. Divide the middle point value (from step 2) by 2. This will be the value between the start value and the middle value.
4. Take the values from step 2 and step 3 and add them together. This is your point between the middle point and the end point.

# Lesson 53: Graphing Trig Functions

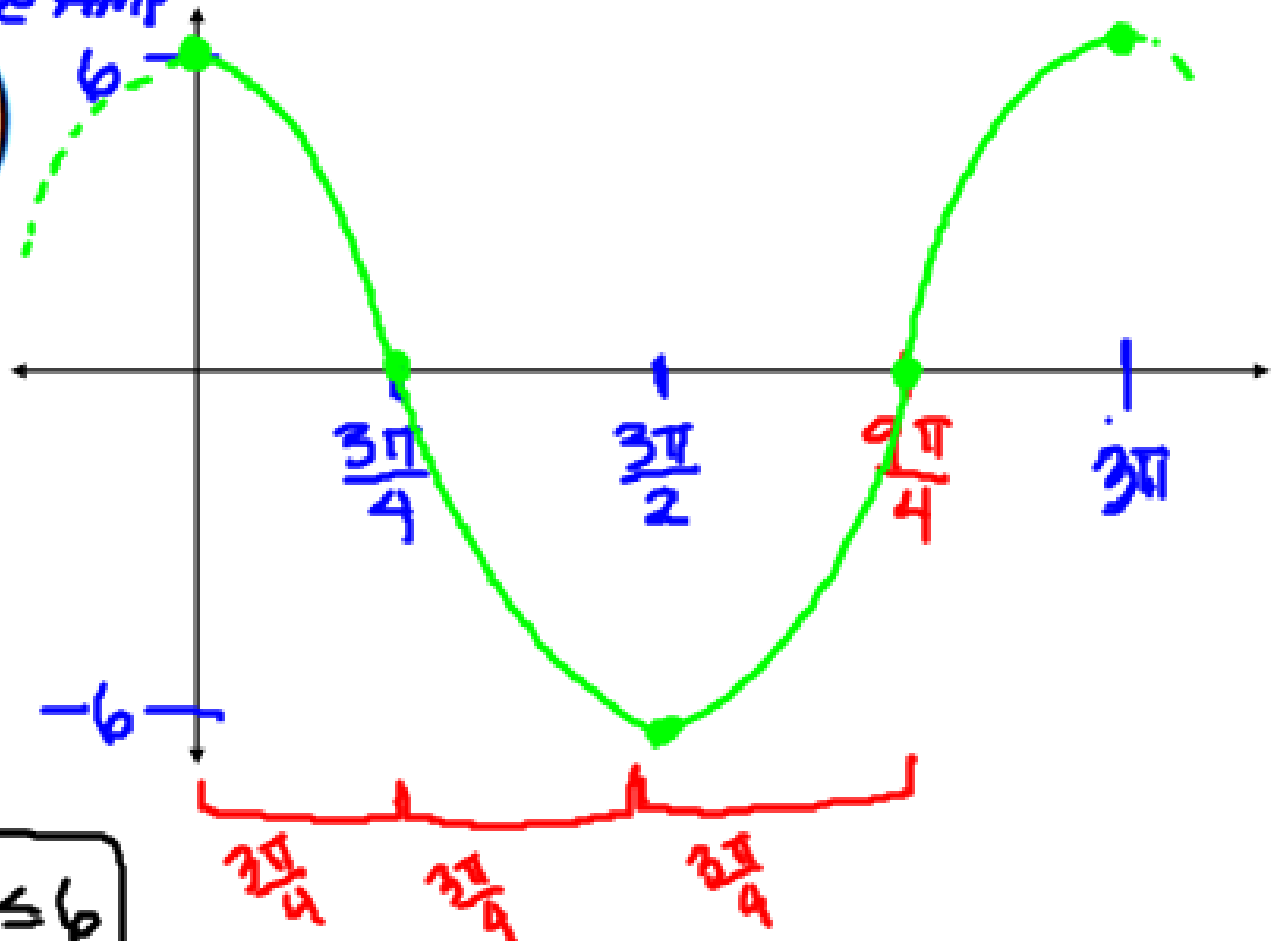
**Example 3:** State the Amplitude and Period of each function.
Now Graph! State the Domain and Range.

c.) $y = 6\cos\left(\frac{2}{3}\theta\right)$

Start @ Amp

$A = 6$

$P = 3\pi$




$\frac{\frac{3\pi}{2}}{2} = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

D: $\mathbb{R}$ R: $-6 \le y \le 6$

$\frac{3\pi}{2} + \frac{3\pi}{4} = \frac{6\pi}{4} + \frac{3\pi}{4} = \frac{9\pi}{4}$

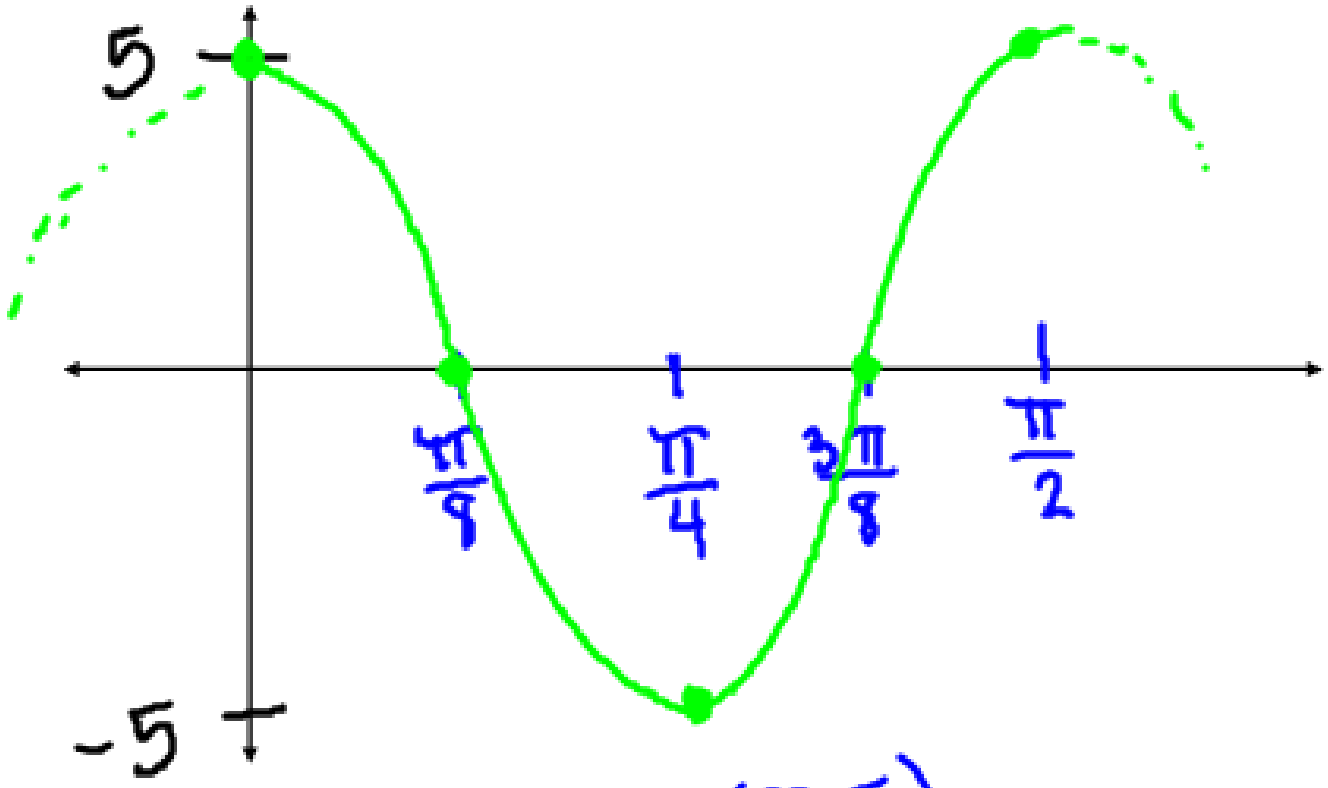
# Lesson 53: Graphing Trig Functions

$\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$ $\frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$

**Example 4:** Graph the following trig function.

start on amp.

$$y = 5\cos 4\theta$$




Amplitude: 5

Period: $\frac{\pi}{2}$ $\frac{2\pi}{4}$

Domain: $\mathbb{R}$

Range: $-5 \le y \le 5$

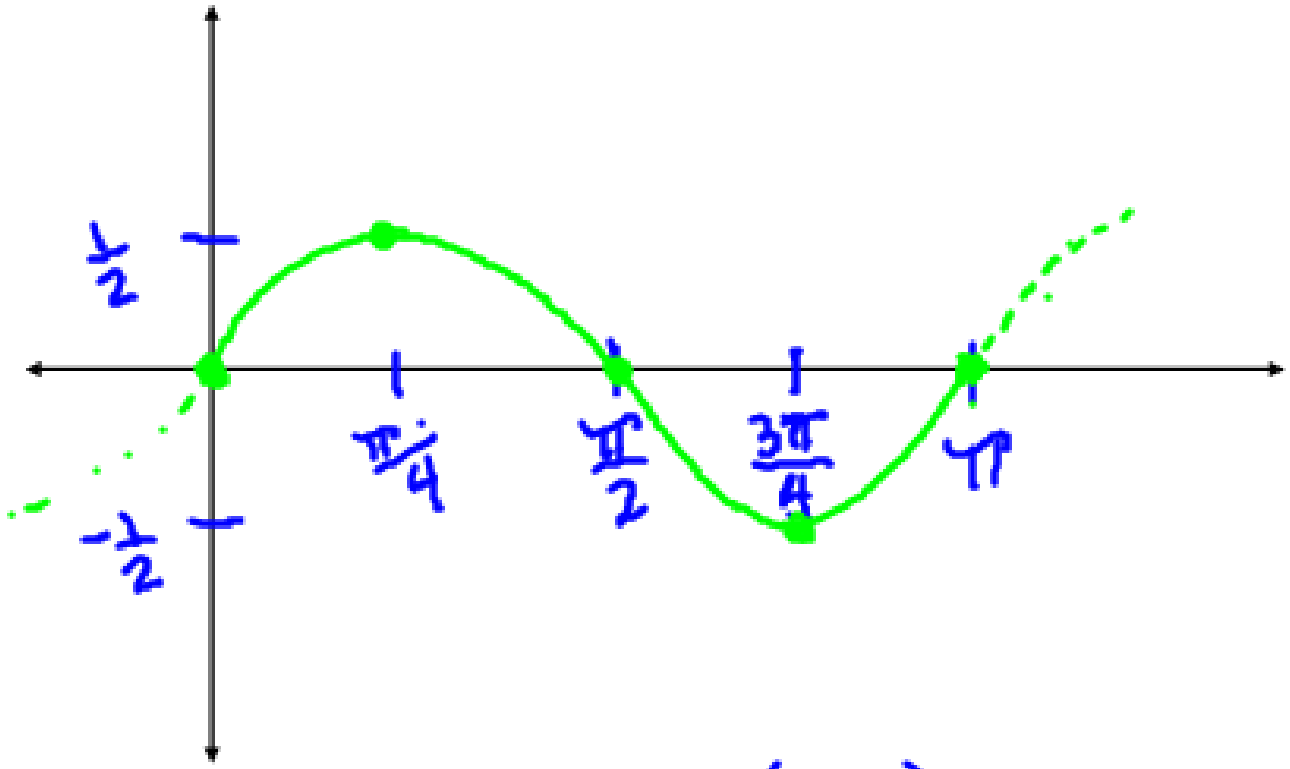
Critical Values: $(0,5)\left(\frac{\pi}{8},0\right)\left(\frac{\pi}{4},-5\right)\left(\frac{3\pi}{8},0\right)\left(\frac{\pi}{2},5\right)$

# Lesson 53: Graphing Trig Functions

**Example 5:** Graph the following trig function.

start at 0.

$$y = \frac{1}{2}\sin 2\theta$$




Amplitude: $\frac{1}{2}$

Period: $\pi$

$\frac{2\pi}{2} = \pi$

Domain: $\mathbb{R}$

Range: $-\frac{1}{2} \le y \le \frac{1}{2}$

Critical Values: $(0,0)\left(\frac{\pi}{4}, \frac{1}{2}\right)\left(\frac{\pi}{2}, 0\right)\left(\frac{3\pi}{4}, -\frac{1}{2}\right)(\pi, 0)$

# Lesson 53: Graphing Trig Functions

## By the end of the lesson, we will be able to:

~Find the Amplitude and Period
~Graph Sine and Cosine functions

Can you?

Lesson 53: Graphing Trig Functions

# Homework:

Assignment 53