Lesson 54

Example 1: Draw a tree
Suppose your state is adding a new area code. The first digit must be a 6 or a 7, the second digit must be a 0 or 1, and the third digit can be 3, 4, or 5. How many area codes are possible?

The Counting Principle: If we have an event - K - that can happen in k ways and an independent event - M – that can happen in m ways, then event K followed by event M can occur in (k x m) ways.

- Independent Event: One choice does not affect another choice.
- Dependent Event: One choice does affect another choice.

Example 2: Use the Counting Principle
How many area codes are possible if we limit the digits as follows? First: #'s 2-9, Second: #'s 0, 1 and Third: #'s 0-9.

Example 3: Use the Counting Principle
How many four-letter patterns can be formed using the letters A, B, C, and D if each letter is used exactly once?

Example 4: Use the Counting Principle
How many two-digit numbers can be formed from the digits 1, 2, 3, 4, and 5 if repetitions are allowed?

Example 5:
Find out how many ways 9 different books can be arranged on a shelf. (This is a DEPENDENT EVENT because the book we pick depends on what other books we have picked previously...)

Example 6:
How many different ways can 5 cars be parked along the street if the only red one must be in the middle? (This is a DEPENDENT EVENT.)

Factorial: Uses the symbol "!". It means to multiply by each integer less than the number.

- For example: $4! = 4 \cdot 3 \cdot 2 \cdot 1$

  Calculator: MATH $\rightarrow$ PRB $\downarrow$ 4:!

Example 7: Evaluate each expression.

a.) 6!

b.) 10!

c.) 2!

d.) 1!

e.) 0!

Permutations: $P(n,r) = \dfrac{n!}{(n-r)!} = {}_nP_r$ ***Use when order is important***

- *Calculator:* MATH $\rightarrow$ PRB $\downarrow$ 2: nPr

Example 8: Permutation

A group of 5 teens went to the movie theater. They found a row with 7 empty seats. How many different ways can the teens be seated in the row? We can use the Counting Principle to help us.
(Hint: find the number of permutations (order) of 7 seats, taken 5 at a time.)

__ __ __ __ __

Now, use the calculator:

Example 9:

How many ways can 3 books be placed on a shelf if they are chosen from a selection of 8 books?

Distinguishable Permutations - permutations with repeated elements (p): $\frac{n!}{p!}$

Example 10: How many ways can we arrange the letters in the word BANANA?
(Hint: Divide by the Factorial of the letters that repeat.)

Example 11:

How many ways can we arrange the letters in the word PERPENDICULAR?

Lesson 55

Combinations: $C(n,r) = \frac{n!}{(n-r)!r!} = {}_nC_r$ ** Order does NOT matter! **

- *Calculator:* MATH $\rightarrow$ PRB $\downarrow$ 3: nCr

Example 1: Use the definition of Combinations to simplify. (no calculator)

a.) C(5,3) b.) C(9, 6)

Example 1 continued: Use a calculator to simplify.

a.) C(5,3) b.) C(9, 6)

Example 2: Combinations

An Algebra 2 class has 27 students. We want to make a committee of 3 students to plan a party. How many different ways can we do this?

Example 3: Combinations

Subzero has 9 different flavors to put in your ice cream. You can choose 3 flavors to put in it. How many different flavor combinations can you create?

Example 4: Combinations

A basket contains 4 acorn squash, 5 gourds, and 8 pumpkins. How many ways can 2 acorn squash, 1 gourd, and 2 pumpkins be chosen? (Hint: We need 3 different combinations and then multiply them together...)

Example 5: Combinations

A bag contains 8 green marbles, 6 blue marbles, and 9 red marbles. How many ways can 6 marbles be selected to meet the following condition: All Marbles are red.

Example 6: Combinations
A bag contains 8 green marbles, 6 blue marbles, and 9 red marbles. How many ways can 6 marbles be selected to meet the following condition: 2 are blue and 4 are red.

Are the following permutations or combinations?

Example 7: arrangement of 10 books on a shelf.

Example 8: selection of a committee of 3 from 10 people.

Example 9: a hand of 6 cards from a deck of 52 cards.

Example 10: number of ways to make a license plate with 6 numbers without repeating numbers.

Example 11: Combinations
Use the definition of Combinations to simplify.

a.) C(10, 3) b.) C(10, 7)

Do you notice any pattern...?

Example 12: Combinations Solve for n.

a.) C(n,8)=C(n,3) b.) C(30, n)=C(30,18)

Lesson 56

Discrete Random Variable: variables or outcomes to a phenomenon where *all possible outcomes* can be counted (discrete) and there is no outcome that is systematically chosen over other possible outcomes (random).

Example A: List the possible outcomes for the following discrete random variables.

Rolling a fair, six sided die. ______________________________

Flipping a fair coin. _______________________________

Picking two colored beads from a bag contain three beads (red, white, and blue) with replacement before the second pick. ____

Probability is a measure of the likeliness of an event. Probability is defined as the ratio of the number of successes to the total number of possible outcomes. It can be expressed as a fraction, decimal, or percent.

Definition of Probability: $P(success) = \frac{number\ of\ ways\ for\ success}{total\ possible\ outcomes}$

When solving problems with probabilities, always express your answer in a fraction (reduced), a decimal (rounded to 4 decimal places), and a percent (rounded to 2 decimal places.

Example B: A bag of candy contains 12 red, 11 yellow, 5 green, 6 orange, 5 blue, and 16 brown candies.

a.) What is the probability that you will randomly draw a yellow candy from the bag?

b.) What is the probability that you will NOT draw an orange candy from the bag?

Example C: Suppose 3 letters are selected from the word ARRANGEMENTS. Find the probability of randomly selecting three consonants.

In this situation, it would take too long to list all the outcomes. First, find how many 3 letter groups meet the condition. There are 8 consonants in the word, and we need a group of 3. Since order doesn't matter, use a combination: _______________

Next, find the total number of ways to choose 3 letters from the word. There are 12 letters in the word, and we are selecting a group of 3. _______________

Then find the ratio of successes to total outcomes: ______________

Example D: The committee to organize the school prom has 6 seniors and 5 juniors. A subcommittee of 4 students is selected at random to choose the music for the prom. What is the probability that the subcommittee will contain 2 seniors and 2 juniors?

- Two events are independent if the outcome of one event has no impact on the outcome of the second event. (Independent events are sometimes called unconditional.)
  - Probability of Independent Events: $P(A\ and\ B) = P(A) \cdot P(B)$
- Two events are dependent is the outcome of one event affects the outcome of the other event. When calculating the probability of the second event, you assume that the first event did occur. (Dependent events are sometimes called conditional.)
  - Probability of Dependent Events: $P(A\ and\ B) = P(A) \cdot P(B\ following\ A)$

Example E: Two dice are rolled. Find the probability of each:

a.) P(1 and 5)=

b.) P(even and odd)=

c.) P(two different #'s)=

Example F: There are 3 quarters, 4 dimes, and 5 nickels in a purse. Suppose 3 coins are to be selected without replacement. Find the probability that you will select a quarter, then a dime, then a nickel.
$P(Q \text{ } and \text{ } then \text{ } D \text{ } and \text{ } then \text{ } N) =$

When talking about probability, a lot of examples use decks of cards. So, here is what is contained in a deck of cards.

- There are 52 cards in a deck.
- 26 are red, 26 are black
- There are 4 suits with 13 cards each: clubs (black), spades (black), hearts (red), diamonds (red).
- Each suit contains: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace (1)

Example G: From a deck of 52 cards, 3 cards are randomly chosen. They are a 10, jack, and another 10, in that order.

a.) Find the probability that this event occurs if each card is replaced each time.

b.) Find the probability that this event occurs if each card is NOT replaced each time.