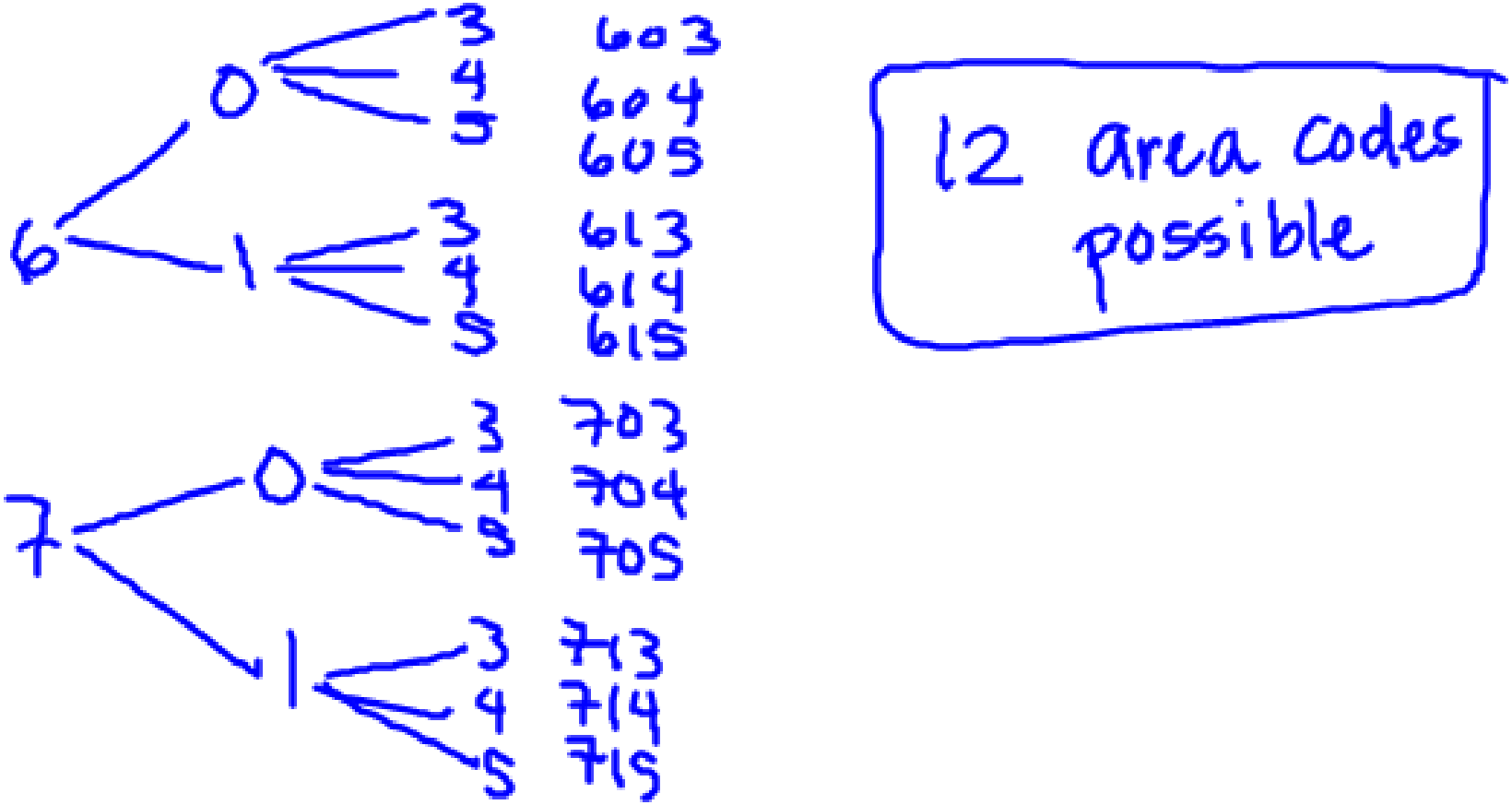
Lesson 54 - Counting and Permutations

Objectives:

~ Use the Fundamental Counting Principle
~ Calculate Permutations

Example 1: Draw a tree

Suppose your state is adding a new area code. The first digit must be a 6 or a 7, the second digit must be a 0 or 1, and the third digit can be 3, 4, or 5. How many area codes are possible? ___ ___ ___



The Counting Principle

If we have an event - K - that can happen in k ways and an independent event - M - that can happen in m ways,

Then event K followed by event M can occur in (k x m) ways.

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Definitions:

Independent Event: One choice does not affect another choice.

Dependent Event: One choice does affect another choice.

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Indep.

Example 2: Use the Counting Principle

How many area codes are possible if we limit the digits as follows? First: #'s 2-9, Second: #'s 0, 1 and Third: #'s 0-9.

8 2 10

$8 \cdot 2 \cdot 10 =$ 160 ways

dep.

Example 3: Use the Counting Principle

How many four-letter patterns can be formed using the letters A, B, C, and D if each letter is used exactly once?

$4 \cdot 3 \cdot 2 \cdot 1 =$ 24 patterns

indep.

Example 4: Use the Counting Principle

How many two-digit numbers can be formed from the digits 1, 2, 3, 4, and 5 if repetitions are allowed?

$5 \cdot 5 =$ 25 two digit #'s

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Example 5:

Let's help Peppermint Patty overcome her math anxiety. Find out how many ways 9 different books can be arranged on a shelf. (This is a DEPENDENT EVENT because the book we pick depends on what other books we have picked previously...)

$$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= \boxed{362,880 \text{ ways}}$$

Example 6:

How many different ways can 5 cars be parked along the street if the only red one must be in the middle? (This is a DEPENDENT EVENT.)

$4 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 24$ ways

↑ only red car

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Ever wondered what the "!" means in math? Well, now we are going to find out.

Factorial uses the symbol "!" and means to multiply by each integer less than the number.

For example, $4!$ means $4 \cdot 3 \cdot 2 \cdot 1$.

Calculator: MATH → PRB ↓ 4:!

Book: $9!$

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Example 7: Evaluate each expression.

a.) $6! = 720$

b.) $10! = 3628800$

c.) $2! = 2$

d.) $1! = 1$

e.) $0! = 1$ ← definition

Permutations

When a group of objects is arranged in a certain order, the arrangement is called a PERMUTATION.

definition** Order matters! ** **

Book not. — calc. not.

Permutations: $$P(n,r) = \frac{n!}{(n-r)!} = {}_nP_r$$

The n is the total number and the r is how many we need to order.

Example 8: Permutation

A group of 5 teens went to the movie theater. They found a row with 7 empty seats. How many different ways can the teens be seated in the row?

We can use the Counting Principle to help us. (Hint: find the number of permutations (order) of 7 seats, taken 5 at a time.)

Counting Princ.

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$ ways

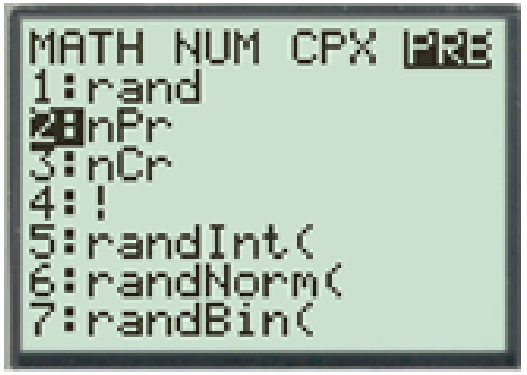
Perm.

$$P(7,5) = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$$

$= 2520$ ways

$${}_nP_r = {}_7P_5$$

Calculators can help us out solving Permutations.



1. Go to "Math"
2. Then arrow over to "Prob".
3. Find the "nPr"

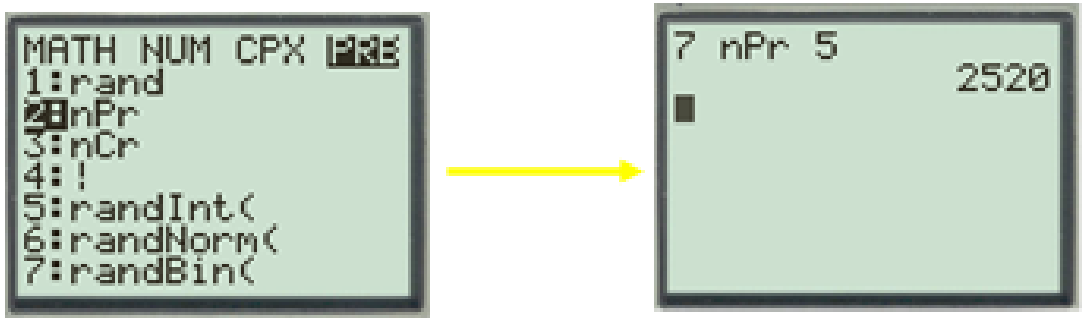
The n is the total number and the r is how many we need to order.

Example 8: Permutation - use the Calculator

A group of 5 teens went to the movie theater. They found a row with 7 empty seats. How many different ways can the teens be seated in the row?

Remember nPr ~~~ n is 7 and r is 5. P(7,5).

$P(7,5) =$ 2520 ways



Example 9:

How many ways can 3 books be placed on a shelf if they are chosen from a selection of 8 books?

$P(8,3) =$ 8 nPr 3 = 336 ways (calc)

Def:

$$P(8,3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot 7\cdot 6\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 8\cdot 7\cdot 6$$

$= 336$ ways

Counting Princ.

$8 \cdot 7 \cdot 6 =$ 336 ways

Distiguishable Permutations

n = # of letters

Permutations with repeated elements: $\frac{n!}{p!}$

p = # of times a letter repeats

Example 10:

How many ways can we arrange the letters in the word BANANA? 6 letters

(Hint: Divide by the Factorial of the letters that repeat.)

Repeats

A: 3

N: 2

$$\frac{6!}{2!\,3!} = \frac{6!}{(2! * 3!)} = \boxed{60 \text{ ways}}$$

Example 11:

How many ways can we arrange the letters in the word PERPENDICULAR? 13 letters

(Hint: Divide by the Factorial of the letters that repeat.)

P: 2

E: 2

R: 2

$$\frac{13!}{2!\ 2!\ 2!} = \frac{13!}{(2! * 2! * 2!)} =$$

778,377,600 ways

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Objectives:

~ Use the Fundamental Counting Principle
~ Calculate Permutations

Can you?

Homework:

Assignment 54

#11 - just solve - ignore the instructions

AND

Review for Test 13

Due next time