Lesson 55: Combinations

Objectives:

~ Calculate Combinations
~ Distinguish between Perm. and Comb.

Combinations

When a group of objects is arranged and order does NOT matter, it is called a Combination.

★ ** Order does NOT matter! **

Book → $C(n,r) = \frac{n!}{(n-r)!r!}$ ← definition

Combinations: $= {}_nC_r$ ← calc

$\binom{n}{r}$ ← not going to use

The n is the total number and the r is how many we need to order.

Lesson 55: Combinations

$$\frac{n!}{(n-r)!\,r!}$$

Example 1: Combinations

Use the definition of Combinations to simplify.

a.) $C(5,3)$

$$= \frac{5!}{(5-3)!\,3!}$$

$$= \frac{5!}{2!\,3!}$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$$

$$= 5 \cdot 2$$

$$= \boxed{10}$$

b.) $C(9,6)$

$$= \frac{9!}{(9-6)!\,6!}$$

$$= \frac{9!}{3!\,6!}$$

$$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 3 \cdot 4 \cdot 7 = 12 \cdot 7$$

$$= \boxed{84}$$

Lesson 55: Combinations

Example 1: Combinations

Calculator: MATH → PRB ↓ 3: nCr

a.) C(5,3)

5 nCr 3

= 10

b.) C(9, 6)

9 nCr 6

= 84

Permutations - order matters

Presidency

Combinations - order doesn't matter

Committee

Example 2: Combinations

Comb

An Alg 2 class has 27 students. We want to make a committee of 3 students to plan a party. How many different ways can we do this?

$C(27,3)$

calc: 27 nCr 3

$= 2925$

def: $\frac{27!}{(27-3)!\,3!} = \frac{27!}{(24!\,3!)}$

$C(27,3) = 2925$ ways

Lesson 55: Combinations

Example 3: Combinations

Subzero has 9 different flavors to put in your ice cream. You can choose 3 flavors to put in it. How many different flavor combinations can you create?

$C(9,3) =$ 84 combinations

$9\ nCr\ 3 = 84$

def: $\frac{9!}{(9-3)!\,3!} = \frac{9!}{6!\,3!}$

Lesson 55: Combinations

Example 4: Combinations

A basket contains 4 acorn squash, 5 gourds, and 8 pumpkins. How many ways can 2 acorn squash, 1 gourd, and 2 pumpkins be chosen? (Hint: We need 3 different combinations and then multiply them together...)

squash · gourds · pumpkin

$C(4,2) \cdot C(5,1) \cdot C(8,2)$

$= 4nCr2 \cdot 5nCr1 \cdot 8nCr2$

$= 6 \cdot 5 \cdot 28$

$=$ **840 ways**

Example 5: Combinations

A bag contains 8 green marbles, 6 blue marbles, and 9 red marbles. How many ways can 6 marbles be selected to meet the following condition: All Marbles are red.

$C(9,6) = 84$ ways

9 nCr 6

Example 6: Combinations

A bag contains 8 green marbles, 6 blue marbles, and 9 red marbles. How many ways can 6 marbles be selected to meet the following condition: 2 are blue and 4 are red.

Blue · red

$C(6,2) \cdot C(9,4)$

$= 6nCr2 \cdot 9nCr4$

$= 15 \cdot 126$

$= 1890$ ways

Permutations

Or

Combinations

Lesson 55: Combinations

Example 7:

Arrangement of 10 books on a shelf.

↳ order matters

Permutation

Lesson 55: Combinations

Example 8:

Selection of a committee of 3 from 10 people.

↳ order doesn't matter

Combination

Lesson 55: Combinations

Example 9:

A hand of 6 cards from a deck of 52 cards.

↳ order doesn't matter

Comb.

Example 10:

Number of ways to make a license plate with 6 numbers without repeating numbers.

order matters

Permutation

(Perm.)

Example 11: Combinations

Use the definition of Combinations to simplify.

a.) $C(10, 3) = \frac{10!}{(10-3)!\,3!}$

$= \frac{10!}{7! \cdot 3!}$

$= 120$

b.) $C(10, 7) = \frac{10!}{(10-7)!\,7!}$

$= \frac{10!}{3! \cdot 7!}$

$= 120$

Same

Do you notice any pattern...?

Our "r"-end #'s - (3+7) have to add to be our "n"-total-(10)

Lesson 55: Combinations

Example 12: Combinations

Solve for n.

two answers

a.) $C(n,8)=C(n,3)$

$8+3=n$

$n=11$

b.) $C(30, n)=C(30,18)$

$n=18$

$30=n+18$

$n=12$

Lesson 55: Combinations

Objectives:

~ Calculate Combinations
~ Distinguish between Perm. and Comb.

Can you?

Homework:

Assignment 55