Objectives:

- Compute Absolute Values of Real Numbers.
- Add, Subtract, Multiply and Divide Signed Numbers.
- Perform Operations of Fractions.
- Know the Associative and Distributive Properties of Real Numbers.

OPERATIONS ON NUMBERS:

$+$	Addition, sum, plus
$-$	Subtraction, difference, minus
$\times, *, \cdot$	Multiplication, product, times.
	Also seen as: ab, $(a)(b)$ Each of the values being multiplied is called a "factor".
$\div$	Division, quotient, divided by.
	Also seen as a fraction $\frac{a}{b}$.

ABSOLUTE VALUE:

By definition, an absolute value is the distance of a number from the zero (0) on a number line. Since distance is always positive, the absolute value bars will make whatever is inside equal to a positive value.

$$|a| = a, \quad |-a| = a$$

Always reduce the quantity inside the absolute value bars first! However – nothing can go inside the bars that starts out on the outside, so don't distribute outside factors!

SECTION R.3 - Operations on Signed Numbers and Properties of Real Numbers

Examples:

A) $|3| =$ 3

B) $|-3| =$ 3

C) $|5-7| =$ $|-2| = 2$

D) $-5|-4| =$ $-5(4) = -20$

RULES and PROPERTIES FOR ADDING & SUBTRACTING:

~ 2 POSITIVE numbers: add or subtract as specified.

(Example: $2 + 2 = 4$, $2 + (-2) = 2 - 2 = 0$)

~ 2 NEGATIVE numbers:

ADDING- add the numbers but keep the negative sign.

(Example: $-4 + (-6) = -(4+6) = -10$)

SUBTRACTING- switch the sign of the second and add .

(Example: $-2 + (+3) = -2 + 3 = 1$)

~ 1 POSITIVE and 1 NEGATIVE : Subtract the smallest from the largest regardless of sign, then put the sign of the larger value in front of the answer.

(Example: $6 - 4 = 2$, $4 - 6 = -2$ because $6-4 = 2$, but 6 is larger)

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RULES and PROPERTIES FOR ADDING & SUBTRACTING:

- COMMUTATIVE PROPERTY OF ADDITION: says that you can move values around. $a + b = b + a$ Make sure you move the sign along with the number. ex: $6 - 4 = -4 + 6$
- ADDITIVE INVERSE: for any number a, there is a $-a$ such that $a + (-a) = 0$. This means that if you add the opposite of any number to itself, you will get 0. The Additive inverse of 5 is -5. The additive inverse of -4 is -(-4) or 4.
- ADDITIVE IDENTITY: the additive Identity for any number is *always* 0 (zero). $a + 0 = 0 + a = a$
- ASSOCIATIVE PROPERTY FOR ADDITION: parentheses can be moved or dropped with addition.

$$(a + b) + c = a + (b + c) = a + b + c$$

RULES and PROPERTIES FOR MULTIPLICATION:

- SAME SIGNS: the product will always be positive!
  (+)(+)=(+), (–)(–)=(+)
- DIFFERENT SIGNS: the product will always be negative!
  (+)(–)=(–), (–)(+)=(–)
- COMMUTATIVE PROPERTY OF MULTIPLICATION:
  says you can move factors around. $a \cdot b = b \cdot a$
- MULTIPLICATIVE INVERSE: also called the *RECIPROCAL*, says that for any number a, there is a value $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right) = \frac{1}{a}(a) = 1, \quad a \neq 0$. So the reciprocal of 5 is 1/5, and the reciprocal of ½ is 2.

RULES and PROPERTIES FOR MULTIPLICATION:

- **MULTIPLICATIVE IDENTITY:** The multiplicative identity of any number a is ***always 1.*** $a(1) = 1(a) = a$
- **ASSOCIATIVE PROPERTY FOR MULTIPLICATION:**
  $(a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot b \cdot c$
- **DISTRIBUTIVE PROPERTIES OF MULTIPLICATION:**
  You can multiply everything within a set of parentheses by the value outside the parentheses.

$$a(b + c) = ab + ac \qquad (a + b)c = ac + bc$$

Examples: Find the Additive Inverse and the Multiplicative Inverse of the following:

E) 7

add: -7

mult: $\frac{1}{7}$

F) -3

add: 3

mult: $-\frac{1}{3}$

G) -4/5

add: $\frac{4}{5}$

mult: $-\frac{5}{4}$

Examples: Use the Distributive property to simplify the following.

H) $(n-3)2$

$= 2n-6$

I) $-\frac{1}{3}(6x-8) = -\frac{1}{3}\cdot\frac{6x}{1} + \frac{1}{3}\cdot\frac{8}{1}$

$= -2x + \frac{8}{3}$

RULES & PROPERTIES FOR DIVISION:

- SAME SIGNS: The answer will be positive.

$$\frac{(+)}{(+)} = +, \quad \frac{(-)}{(-)} = +, \quad \frac{-a}{-b} = \frac{a}{b}$$

- DIFFERENT SIGNS: The answer will be negative.

$$\frac{(+)}{(-)} = -, \quad \frac{(-)}{(+)} = -, \quad -\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$$

- DIVISION PROPERTIES:
  - $\frac{0}{a} = 0$ zero divided by any number (excepting zero again) equals zero.
  - $\frac{a}{a} = 1$ any number divided by itself equals 1.
  - $\frac{a}{0}$ is ***UNDEFINED*** when $a \neq 0$. Why? Say $x = \frac{2}{0}$, then $0 \cdot x = \frac{2}{0} \cdot 0$, $0 = 2$? Not possible, so it's undefined.

RULES & PROPERTIES FOR DIVISION: (Cont.)

- DIVISION PROPERTIES:
  - $\frac{0}{0} \neq 0$ it is ***INDETERMINATE*** instead. Why?

    Say $x = \frac{0}{0}$, then $0 \cdot x = \frac{0}{0} \cdot 0$, $0 \cdot x = 0$.

    x could equal anything and the equation would still be true, so we can't really say it's always equal to zero.

- REDUCTION PROPERTY: if a, b, c are real numbers, then $\frac{ac}{bc} = \frac{a}{b}$ *if* $b \neq 0, c \neq 0$. If you can factor out common factors in the numerator and denominator (remember that 1 is always a factor), you can set them equal to 1 (sometimes called "cancelling") and reduce the fraction.

SECTION R.3 - Operations on Signed Numbers and Properties of Real Numbers

Examples: Reduce the following:

J) $\frac{14}{35} = \frac{7 \cdot 2}{7 \cdot 5} = \boxed{\frac{2}{5}}$

K) $\frac{40}{15} = \frac{5 \cdot 8}{5 \cdot 3} = \boxed{\frac{8}{3}}$

L) $\frac{7}{42} = \frac{7 \cdot 1}{7 \cdot 6} = \boxed{\frac{1}{6}}$

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WORKING WITH FRACTIONS (Operators):

- $\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}, \quad c \neq 0, \ d \neq 0$

- $\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \cdot \frac{d}{b} = \frac{ad}{cb}, \quad b, c, d \neq 0$ Invert the 2nd fraction, then multiply.

- $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}, \quad c \neq 0$ You must have common denominators to add fractions.

- $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{cb}{bd} = \frac{ad+cb}{bd}, \quad b, d \neq 0$ Find common denominators before adding!

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Examples:

M) $\frac{15}{12} \cdot \frac{9}{25} = \frac{3}{4} \cdot \frac{3}{5} = \boxed{\frac{3}{20}}$

N) $-\frac{10}{3} \div \frac{15}{7} = -\frac{10}{3} \cdot \frac{7}{15} = \boxed{\frac{-14}{9}}$

O) $\frac{9}{4} - \frac{3}{4} = \frac{6}{4} = \frac{2 \cdot 3}{2 \cdot 2} = \boxed{\frac{3}{2}}$

P) $\frac{7 \cdot 3}{12 \cdot 3} + \frac{1 \cdot 4}{9 \cdot 4} = \frac{21}{36} + \frac{4}{36} = \boxed{\frac{25}{36}}$

FINDING THE LEAST COMMON DENOMINATOR (LCD):

1. Factor each denominator.
2. List all of the factors that are contained in the fractions. If a factor appears in more than one denominator (these are the "common" factors), write it only once! If a factor appears more than once in a single denominator, write it as many times as it appears. The product of this list of factors is your Least Common Denominator (LCD).
3. To Add or Subtract, figure out what each of your fraction denominators are missing (we want them to all look the same as the LCD), then multiply the fraction - TOP & BOTTOM BOTH - by the missing factors. (This is like multiplying the fraction by 1 - think the opposite of the Reduction property.)

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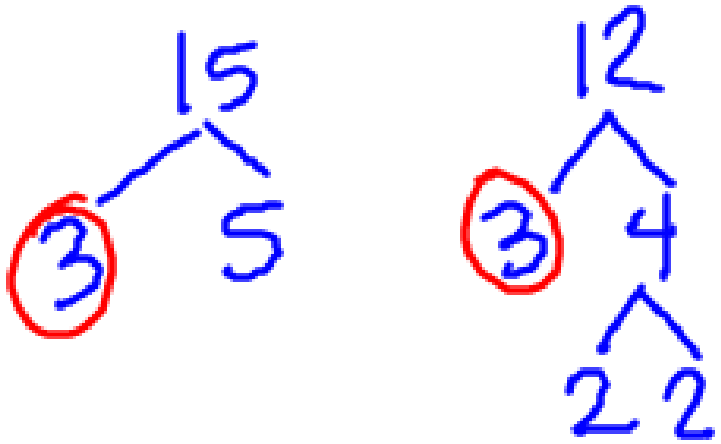
Examples:

Q) Find the LCD of $\frac{8}{15}$ and $\frac{5}{12}$.



R) Add $\frac{7}{12}+\frac{1}{9}$

C: 3

U: 5 · 2 · 2

LCD: 3 · 5 · 2 · 2

= 60

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Can You?

Homework: Section R.3:

Pg. 31: #8, 10-17 all, 29, 32, 33, 40, 41, 42, 44, 48, 51-75 odds, 80-86 evens, 89, 90
(36 problems)